Math 242 Merit Fall 2005 - Worksheet 16

Linear Approximation

1. Suppose $f_x(3,4) = 5$, $f_y(3,4) = -2$, and $f(3,4) = 6$. Assuming the function is differentiable, what is the best estimate for $f(3.1, 3.9)$ using this information?

2. Use differentials to approximate $\sqrt{26}\sqrt[3]{28}\sqrt[4]{17}$.

3. Use differentials to approximate $e^{0.4} = e^{(1.1)^2-(0.9)^2}$.

   Now do this problem by finding the tangent plane to a surface.

4. The base radius r and the height h of a right circular cylinder are measured as 3 cm and 9 cm, respectively. There is a possible error of 1 mm in each measurement. Use differentials to estimate the maximum possible error in computing:

   (a) the volume of the cylinder;

   (b) the total surface area of the cylinder.

5. Four positive numbers, each less than 50 are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum possible error in the computed product that might result from the rounding.

6. The range of a cannonball with initial velocity v_0 and inclination angle α from the horizontal is $R = \frac{1}{32}v_0^2 \sin 2\alpha$. Use differentials to estimate the change in range if v_0 is increased from 400 to 410 ft/s and α is increased from 30° to 31°. Now compare ΔR with $\mathrm{d}R$.

7. Find $\frac{\mathrm{d}w}{\mathrm{d}t}$ both by using the chain rule and by expressing w as a function of t before differentiating, where

$$w = e^{-x^2-y^2}; \qquad x = t; \qquad y = \sqrt{t}$$

Warm-up for Thursday

Read Section 12.7. Find $\frac{\mathrm{d}w}{\mathrm{d}t}$ both by using the chain rule and by expressing w as a function of t before differentiating, where

$$w = \frac{1}{u^2 + v^2}; \qquad u = \cos 2t; \qquad v = \sin 2t$$