Math 242 Merit Fall 2005 - Worksheet 17

The Chain Rule

1. A company sells regular widgets for \$4 apiece and premium widgets for \$6 apiece. If the demand for regular widgets is growing at a rate of 200 widgets per year and the demand for premium widgets is dropping at the rate of 80 per year, the company's revenue is

   (a) staying constant;

   (b) increasing;

   (c) decreasing.

2. A particle Q moving through space is being studied. Let s denote the distance that Q has travelled with respect to some starting point. (We can think of distance as arc length on the curve determined by Q.)

   (a) We know that s depends on two factors X and Y.

   (b) We know that X and Y vary over time t in years according to the formulae $X = t^2 - 1$ and $Y = \ln t$.

   (c) The changes in s with respect to X and Y are both constants, a and b respectively.

   What is the speed of the particle Q, in terms of a and b, after 20 years?

3. Suppose $R = f(u, v, w)$, $u = g(x, y, z)$, $v = h(x, y, z)$ and $w = j(x, y, z)$. In the chain rule, how many terms do you have to add up to find the partial derivative with respect to x?

4. Suppose $w = \ln(x^2 + y^2 + z^2)$, where $x = s - t$, $y = s + t$ and $z = 2\sqrt{st}$. Find $\frac{\partial w}{\partial s}$ and $\frac{\partial w}{\partial t}$.

5. If $g(s, t) = f(s^2 - t^2, t^2 - s^2)$ and f is differentiable, show that g satisfies
$$t\frac{\partial g}{\partial s} + s\frac{\partial g}{\partial t} = 0.$$

6. The radius of a right circular cylinder is decreasing at a rate of 1.5 cm/s while its height is increasing at a rate of 4 cm/s. At what rate is the volume of the cylinder changing when the radius is 50 cm and the height is 100 cm?

7. The length l, width w and height h of a box change with time. At a certain instant the dimensions are $l = 1$ m and $w = h = 2$ m, and l and w are increasing at a rate of 2 m/s, while h is decreasing at a rate of 3 m/s. At that instant, find the rates at which the following quantities are changing:

   (a) The volume;

   (b) The surface area;

   (c) The length of a diagonal.

Warm-up for next Tuesday

1. Find ∇f at $P\ (-5, 1, 3)$ for $f(x, y, z) = (2x - 3y + 5z)^5$.

2. Find the directional derivative of $f(x, y, z) = e^{xyz}$ at $P\ (2, -1, -2)$ in the direction of $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{j} - \mathbf{k})$.