

Merit Worksheet 18, Math 242, Fall 2005

1. The volume V (in cubic centimetres) and pressure p (in atmospheres) of n moles of an ideal gas satisfy the equation $pV = nRT$, where T is its temperature (in Kelvins) and $R = 82.06$. Suppose that a sample of the gas has a volume of 10 L when the pressure is 2 atm and the temperature is 300°K . If the pressure is increasing at 1 atm per minute and the temperature is decreasing at $10^\circ\text{K}/\text{min}$, is the volume of the gas decreasing or increasing? At what rate?
2. Calculate the derivative matrix of the spherical coordinate transformation given by $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$, and $z = \rho \cos \phi$. Use this to find $\frac{\partial w}{\partial \rho}$, $\frac{\partial w}{\partial \phi}$ and $\frac{\partial w}{\partial \theta}$ for $w(x, y, z) = x^2 + 3y^2 + 5z^2$.
3. Let $f(x, y, z) = x \cos y + z^2$. Let P be the point $(3, \pi/3, 5)$.
 - (a) Find $\nabla f(P)$.
 - (b) Find the directional derivative of f in the direction \mathbf{j} .
 - (c) Find the directional derivative of f in the direction $\mathbf{i} + \mathbf{j} + \mathbf{k}$.
4. The temperature at a point (x, y) on a metal plate is given by $T(x, y) = 100 - x^2 - 2y^2$. A heat-seeking particle starts at the point $(4, 2)$ and moves at each instant in the direction of maximum temperature increase.
 - (a) In what direction does the particle move initially?
 - (b) Draw some level curves of T in the xy -plane and sketch the path followed by the particle.
 - (c) If you have extra time at the end of the discussion section, then you can write down the equation of this path. (Hint: What is the relationship between the tangent vectors to the curve and the gradient vectors of T ?)
5. What is the maximal value that a directional derivative of $f(x, y) = \frac{2x}{x-y}$ can assume at the point $(3, 1)$?
6. Let f be a function of two variables that has continuous partial derivatives and consider the points $A(1, 3)$, $B(3, 3)$, $C(1, 7)$ and $D(6, 15)$. The

directional derivative of f at A in the direction of B is 3 and the directional derivative of f at A in the direction of D is 9. What is the directional derivative of f at A in the direction of C ?

7. Use the image below to answer these questions: At which point will the gradient vector have the largest magnitude?
- (a) $(0, 4)$
 - (b) $(-4, -4)$
 - (c) $(0, 0)$
 - (d) $(6, -2)$

At which of these points will the gradient vector be most parallel to \mathbf{j} ?

Warm-Up for Next Thursday

Read Section 12.8 - you need to know this material but it will not be covered in lecture. Write a (neat) summary of the important ideas and results from this section.