Merit Worksheet 19, Math 242, Fall 2005

1. Use the image on the other side of this page to answer these questions: At which point will the gradient vector have the largest magnitude?

   (a) $(0, 4)$

   (b) $(-4, -4)$

   (c) $(0, 0)$

   (d) $(6, -2)$

   At which of these points will the gradient vector be most parallel to $\mathbf{j}$?

2. Let $f(x, y, z) = x^2 + y^2 + z^2$. Which statement best describes the vector $\nabla\, g(x, y, z)$ and **why**? It is always perpendicular to:

   (a) a vertical cylinder passing through (x, y, z).

   (b) a horizontal plane passing through (x, y, z).

   (c) a sphere passing through (x, y, z).

   (d) none of the above.

3. Use the gradient vector to write an equation for the line tangent to the curve $x^4 + xy + y^2 - 19 = 0$ at the point $(2, -3)$.

4. The surfaces $x^2y^2 + ax + z^3 = 16$ and $3x^2 + y^2 - 2z = 9$ intersect in a curve that passes through the point $P(2, 1, 2)$. Find a tangent vector to the curve of intersection at P.

5. Find an equation for the plane tangent to the paraboloid $z = 2x^2 + 3y^2$ and, simultaneously, parallel to the plane $4x - 3y - z = 10$.

6. The curve $\mathbf{r}(t) =< t^2/2, 4/t, t/2 - t^2 >$ intersects the surface $x^2 - 4y^2 - 4z = 0$ at the point $(2, -2, 3)$. What is the angle of intersection?

7. Find the point of the surface $z = xy + 1$ that is closest to the origin.

8. Suppose that you are standing at the point with coordinates $(-100, -100, 430)$ on a hill that has the shape of the graph of $z = 500 - (0.003)x^2 - (0.004)y^2$ (in units of metres). In what (horizontal) direction should you move in order to maintain a constant altitude?

Warm-Up for next Tuesday

Read Section 12.9.

Find the maximum and minimum values (if any) of the function $f(x, y) = 2x + y$ subject to the constraint $x^2 + y^2 = 1$.