

Merit Worksheet 20, Math 242, Fall 2005

1. Let $f(x, y) = ax + by + c$ on the domain $x^2 + y^2 \leq 4$. What are the maximum and minimum values of f for the given domain?
2. How many local maxima and minima does the function $f(x, y, z) = ax + by + cz$ have on the sphere $x^2 + y^2 + z^2 = 1$?
3. This contour plot of $f(x, y)$ also shows the circle of radius 2 centred at $(0, 0)$. How many local maxima and minima does $f(x, y)$ have?

4. This plot shows the gradient vectors for a (hidden) function $f(x, y)$ and a linear constraint. Which point is closest to the global minimum of $f(x, y)$ on this constraint? (A, B, C, or D?)
5. Let $T(x, y, z) = 20 + 2x + 2y + z^2$ represent the temperature at each point on the sphere $x^2 + y^2 + z^2 = 21$. Find the extreme temperature on the curve formed by the intersection of the plane $x + y + z = 3$ and the sphere.
 - (a) Solve for the two constraint functions $g(x, y, z) = 0$ and $h(x, y, z) = 0$.
 - (b) Find the gradient vectors of T, g, h and set up the relevant equations.
 - (c) Solve for the five unknowns.

- (d) Determine the maximum and minimum values.
6. Consider a general triangle with sides of length x, y, z and fixed perimeter P . Maximize its area $A = f(x, y, z) = \frac{1}{2}xy \sin \alpha$ subject to the constraints $x + y + z = P$ and $z^2 = x^2 + y^2 - 2xy \cos \alpha$ (the law of cosines). In particular, show that the optimal such triangle is equilateral, with $x = y = z$.
7. Consider the function $f(x, y) = \frac{1}{2}(ax^2 + by^2)$.
- Show that $(0, 0)$ is a critical point.
 - For what values of a and b does f have a maximum at $(0, 0)$?
What does the surface look like?
 - For what values of a and b does f have a minimum at $(0, 0)$?
What does the surface look like?
 - For what values of a and b does f have a saddle point at $(0, 0)$?
What does the surface look like?
8. Which of the following guarantees a saddle point of the function $f(x, y)$ at the point (a, b) ?
- f_{xx} and f_{yy} have the same sign at (a, b)
 - f_{xx} and f_{yy} have different signs at (a, b)
 - f_{xy} is negative at (a, b)
 - None of the above.
9. Which of the following would be enough evidence to conclude that $f(x, y)$ has a global minimum?
- D is always positive
 - $f_{xx} > 0$ and $f_{yy} > 0$
 - $f(x, y)$ has no saddle point and no local maxima
 - None of the above.

Warm-Up for next time: Find and classify the critical points of the function $f(x, y) = x^3 + y^3 + 3xy + 3$.