Merit Worksheet 21, Math 242, Fall 2005

1. Consider the function $f(x, y) = \frac{1}{2}(ax^2 + by^2)$.

   (a) Show that $(0, 0)$ is a critical point.

   (b) For what values of a and b does f have a maximum at $(0, 0)$? What does the surface look like?

   (c) For what values of a and b does f have a minimum at $(0, 0)$? What does the surface look like?

   (d) For what values of a and b does f have a saddle point at $(0, 0)$? What does the surface look like?

2. Which of the following guarantees a saddle point of the function $f(x, y)$ at the point (a, b)?

   (a) f_{xx} and f_{yy} have the same sign at (a, b)

   (b) f_{xx} and f_{yy} have different signs at (a, b)

   (c) f_{xy} is negative at (a, b)

   (d) None of the above.

3. Which of the following would be enough evidence to conclude that $f(x, y)$ has a global minimum?

   (a) D is always positive

   (b) $f_{xx} > 0$ and $f_{yy} > 0$

   (c) $f(x, y)$ has no saddle point and no local maxima

   (d) None of the above.

4. Show that if R is the rectangle $x_0 \leq x \leq x_1, y_0 \leq y \leq y_1$

$$\int\int_R \frac{\partial^2 F(x, y)}{\partial x \partial y} \, \mathrm{d}x \, \mathrm{d}y = F(x_1, y_1) - F(x_0, y_1) - F(x_1, y_0) + F(x_0, y_0)$$

5. Evaluate each of the following iterated integrals.

   (a) $\int_1^3 \int_0^1 (1 + 3x^2 y) \, \mathrm{d}x \, \mathrm{d}y$

   (b) $\int_0^5 \int_0^2 \sin 2x \cos y \, \mathrm{d}x \, \mathrm{d}y$

(c) $\int_0^1 \int_0^1 (x+y)^{-3} \, \mathrm{d}y \, \mathrm{d}x$

(d) $\int_1^2 \int_1^3 e^{x-y} \, \mathrm{d}x \, \mathrm{d}y$

6. Sketch the solid whose volume is given by the iterated integral

$$\int_0^4 \int_0^4 (4 - x^2 - y^2) \, \mathrm{d}y \, \mathrm{d}x.$$