Math 242, Merit Worksheet 22, Fall 2005

1. Find and classify the critical points for the function $f(x, y) = (x^2 + y^2)e^{x^2-y^2}$.

2. Show that the sphere $x^2 + y^2 + z^2 = r^2$ and the cone $z^2 = a^2x^2 + b^2y^2$ are orthogonal (that is, have perpendicular tangent planes) at every point of their intersection.

3. A wire 120 cm long is cut into three pieces. Each piece is then bent into the shape of a square. Let the function f be the sum of the area of these three squares. Show that the single critical point of f is a local minimum. However, it should also be possible to maximise the sum of the areas. Explain.

4. What possible positions could a tangent plane have if $f_x = 0$ at the point of tangency?

5. Sketch the regions of integration and evaluate the following integrals by changing the order of integration:

   (a) $\int_0^1 \int_{y^{1/3}}^1 \frac{1}{\sqrt{1+x^2}}\, dx\, dy$

   (b) $\int_0^8 \int_{x^{2/3}}^4 x \cos y^4\, dy\, dx$

   (c) $\int_0^4 \int_{\sqrt{y}}^2 \frac{ye^{x^2}}{x^3}\, dx\, dy$

6. Find the area in the xy-plane between the curves $y = x$ and $y = 2x^2 - x$. Think about this both as a double integral and a single integral and why they are really the same thing.

7. Find $\int\int_D 3\, dA$ where D is the circular region in the xy-plane with centre $C(10, -50)$ and radius 2. What if the centre is at $C(-1, 15)$?

Warm-Up for Thursday

Evaluate the following iterated integral and sketch the region of integration.

$$\int_0^1 \int_y^1 (x + y)\, dx\, dy$$