

Math 242, Merit Worksheet 23, Fall 2005

1. Find the average value of the function $f(x) = \int_x^1 \cos t^2 dt$ on the interval $[0, 1]$.
2. Evaluate $\int_0^1 \int_0^1 e^{\max(x^2, y^2)} dx dy$, where $\max(x^2, y^2)$ means the larger of the numbers x^2 and y^2 .
3. Without doing any calculations:
 - (a) Let R be the square defined by $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. Is the sign of $\int \int_R x^4 dA$ positive, negative, or zero?
 - (b) What is the value of $\frac{1}{\pi} \int \int_S 1 + x dA$, where S is the unit disk centred at the origin?
4. Find $\int \int_D 3 dA$ where D is the circular region in the xy -plane with centre $C(10, -50)$ and radius 2. What if the centre is at $C(-1, 15)$?
5. Find the area in the xy -plane between the curves $y = x$ and $y = 2x^2 - x$. Think about this both as a double integral and a single integral and why they are really the same thing.
6. Find the volume of the solid T that lies below the paraboloid $z = x^2 + y^2$ and above the triangle R in the xy -plane that has vertices $(0, 0, 0)$, $(1, 1, 0)$, and $(2, 0, 0)$.
7. Describe the solid bounded by the graphs $y = x^2$, $z = y$, $y = 4$ and $z = 0$ and then find its volume.
8. Find the volume of the solid bounded by the two paraboloids $z = x^2 + 2y^2$ and $z = 12 - 2x^2 - y^2$. (There is a picture of this solid on page 962 of your textbook.)

Warm-Up for next Tuesday

Use double integration in polar coordinate to find the area bounded by the cardioid $r = 1 + \cos \theta$.