Math 242, Merit Worksheet 25, Fall 2005

1. Express the following integral in polar coordinates:

$$\int_{-2}^{0}\int_{0}^{\sqrt{4-x^2}} x^2+y^2\ dy\ dx.$$

2. Express the following integral in cartesian coordinates:

$$\int_{0}^{3}\int_{\pi}^{\pi/2} r\ d\theta\ dr$$

3. Evaluate

$$\int_{1}^{2}\int_{0}^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\ dy\ dx$$

4. The Archimedean spiral is the equation $r=\theta$. What area is enclosed by the spiral when θ ranges from 0 to 3π?

5. Find the volume of the "ice-cream cone" bounded by the sphere $x^2+y^2+z^2=1$ and the cone $z=\sqrt{x^2+y^2}$.

6. Find the volume of a sphere of radius a by using double integration.

7. Evaluate

$$\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{(1+x^2+y^2)^2}\ dx\ dy.$$

8. Use double integrals to find the volume of the solid that lies inside the ellipsoid $4x^2+4y^2+z^2=80$ and above the paraboloid $z=2x^2+2y^2$.

9. Use polar coordinates to combine the sum

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\ dy\ dx+\int_{1}^{\sqrt{2}}\int_{0}^{x} xy\ dy\ dx+\int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\ dy\ dx$$

   into one double integral and then evaluate.

10. Use a double integral to evaluate the area enclosed by one leaf of the four-leaved rose $r=\cos 2\theta$.

Warm-Up for the Tuesday after Thanksgiving

Find the centroid of the plane region bounded by the lines $x = 0$, $x = 4$, $y = 0$, $y = 6$. Assume that the density is 2 everywhere in the region.

Have a great break!!