Math 242, Merit Worksheet 26, Fall 2005

1. Use polar coordinates to combine the sum

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\ dy\ dx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xy\ dy\ dx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\ dy\ dx$$

   into one double integral and then evaluate.

2. Use a double integral to evaluate the area enclosed by one leaf of the four-leaved rose $r = \cos 2\theta$.

3. Find the centroid of the rectangular lamina $\{-1 \le x \le 1, -1 \le y \le 1\}$ with density $\delta(x,y) = 2y + x^2$. Can you exploit symmetry in any way?

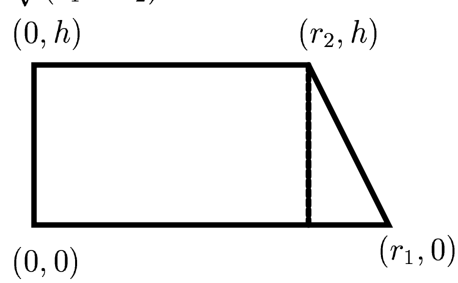
4. Find the mass and centroid of the region bounded by $y = 0$, $x = -1$, $x = 1$, and $y = e^{-x^2}$, with $\delta(x,y) = |xy|$.

5. Use the first theorem of Pappus to find the centroid of the first-quadrant portion of the annular ring with boundary circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2$ (where $0 < a < b$).

6. Apply the second theorem of Pappus to find the centroid of the arc that consists of the first-quadrant portion of the circle $x^2 + y^2 = r^2$.

7. (a) Apply the first theorem of Pappus to show that the volume of the conical frustrum obtained by revolving the trapezoid pictured below around the $y - axis$ is $V = (1/3)\pi h(r_1^2 + r_1 r_2 + r_2^2)$.

   (b) Apply the second theorem of Pappus to show that the lateral surface area of the conical frustrum is $\pi(r_1 + r_2)L$, where $L = \sqrt{(r_1 - r_2)^2 + h^2}$.



Warm-up for Thursday

Compute the value of the triple integral

$$\int_0^2 \int_0^3 \int_0^1 x + y + z \; dz \; dy \; dx \; .$$