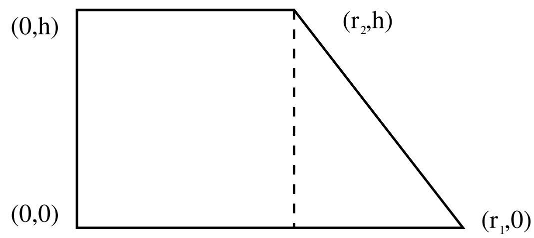
Math 242, Merit Worksheet 27, Fall 2005

1. A conical frustrum is obtained by revolving the trapezoid pictured below around the $y-axis$. Apply the second theorem of Pappus to show that the lateral surface area of the conical frustrum is $\pi(r_1 + r_2)L$, where $L = \sqrt{(r_1 - r_2)^2 + h^2}$.



2. Use Pappus's second theorem to find the surface area obtained when the arc of the semicubical parabola $y^2 = x^3$ between $(1, 1)$ and $(4, 8)$ is rotated around the y-axis.

3. Consider a disc D with density $\rho(x, y) = \frac{1}{\sqrt{x^2+y^2}}$, centred at the origin and of radius a.

   (a) Find the moments of inertia I_x, I_y, I_0.

   (b) If it is rotating at 4 rad/sec., what is KE_{ROT}?

   (c) Where should I place a very small but very dense weight, of the same mass as the disc, so that when rotated about the z-axis, it will have the same kinetic energy as the rotating disc?

4. A uniform rectangular plate with base length a, height b, and mass m is centred at the origin. Show that its polar moment of inertia is $I_0 = \frac{1}{12}m(a^2 + b^2)$.

5. What domain D in space maximizes the value of the integral

$$\int\int\int_D (1 - x^2 - y^2 - z^2)\ dV\ ?$$

6. Consider the integral $\int_{-1}^{1} \int_{x^2}^{1} \int_0^{1-y}\ dz\ dy\ dx$.

   (a) What does this integral evaluate for us?

(b) Sketch the region of integration.

(c) Rewrite the integral in the following orders:

$$dy\ dz\ dx, \quad dx\ dy\ dz, \quad dz\ dx\ dy$$

(d) Evaluate the integral.

Remember: Practice Exam Sunday at 5.30pm in 170 Altgeld