Math 242, Merit Worksheet 28, Fall 2005

1. Consider the plane region bounded by $x^2 = 2py$, $x = 0$ and $y = h = \frac{r^2}{p}$ ($p > 0$). Show that its area is $\frac{2}{3}rh$ and that the x-coordinate of its centroid is $\frac{3}{8}r$. Use Pappus' First Theorem to show that the volume of a paraboloid of revolution with radius r and height h is $V = \frac{1}{2}\pi r^2 h$. Use Pappus' Second Theorem to find its surface area.

2. What domain D in space maximizes the value of the integral

$$\int\int\int_D (1 - x^2 - y^2 - z^2)\, dV \ ?$$

3. Consider the integral $\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y}\ dz\ dy\ dx$.

   (a) What does this integral evaluate for us? (In words)

   (b) Sketch the region of integration.

   (c) Rewrite the integral in the following orders:

$$dy\ dz\ dx, \quad dx\ dy\ dz, \quad dz\ dx\ dy$$

   (d) Evaluate the integral.

4. Set up integrals in rectangular, cylindrical, and spherical coordinates to find the volume of the solid bounded below by the cone $z^2 = x^2 + y^2$ and bounded above by the plane $z = 3$. Which coordinate system seems most promising here?

5. Let E be the solid region bounded by the lower half-cone $z = -2\sqrt{x^2 + y^2}$ and the two spheres $x^2 + y^2 + z^2 = 1$, $x^2 + y^2 + z^2 = 4$. Express the volume of E as a triple integral in spherical coordinates.

6. Let F be the solid region bounded below by the xy-plane, bounded above by the paraboloid $z = 9 - x^2 - y^2$, and on the sides by the cylinder $x^2 + y^2 - 2x = 0$. Express the volume of F as a triple integral in cylindrical coordinates.