Merit Worksheet 4 - Math 242, Fall 2005

The Cross Product

1. Calculate:

   (a) $(-\mathbf{i} + 4\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$.

   (b) $(\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}) \times (3\mathbf{i} + 9\mathbf{j} + 6\mathbf{k})$.

2. **Without calculating** $\underline{a}$ and $\underline{b}$, decide which of $\underline{c}, \underline{d} = \underline{a} \times \underline{b}$:

   (a) $\underline{a} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}, \underline{b} = -2\mathbf{i} + \mathbf{j} + 6\mathbf{k}$,
   $\underline{c} = 19\mathbf{i} - 28\mathbf{j} + 11\mathbf{k}, \underline{d} = -19\mathbf{i} + 28\mathbf{j} - 11\mathbf{k}$.

   (b) $\underline{a} = 2\mathbf{i} + 1\mathbf{j} - \mathbf{k}, \underline{b} = 3\mathbf{i} + 7\mathbf{k}$,
   $\underline{c} = -7\mathbf{i} + 17\mathbf{j} + 3\mathbf{k}, \underline{d} = 7\mathbf{i} - 17\mathbf{j} - 3\mathbf{k}$.

3. The vectors $\underline{a}$ and $\underline{b}$ determine a parallelogram. Show that the area of this parallelogram is $||\underline{a} \times \underline{b}||$.

4. Find the area of the triangle in $\mathbf{R}^3$ determined by the vectors $\underline{a} =< 3, 4, 1 >$ and $\underline{b} =< 5, 2, 7 >$.

5. Playing with the cross product:

   (a) Consider the vector $\underline{c} = \underline{a} \times (\underline{a} \times \underline{b})$. Is $\underline{c} \perp \underline{a}$? Is $\underline{c} \perp \underline{b}$? Either prove or give a counterexample.

   (b) Suppose that $\mathbf{j} \times \underline{a} = \mathbf{i}$. Give two possible solutions for $\underline{a}$ and discuss other possible solutions.

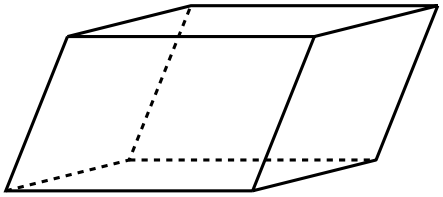
   (c) Given $\underline{a} =< 1, 2, 3 >$ and $\underline{b} =< 1, -1, 1 >$, sketch the collection of all positions vectors $\underline{c}$ satisfying $\underline{a} \times \underline{c} = \underline{a} \times \underline{b}$.

6. Let $\underline{a}$ and $\underline{b}$ be nonzero vectors, θ the angle between them. Complete the following sentence: If $\tan\theta =$ _____, then $\underline{a} \cdot \underline{b} = ||\underline{a} \times \underline{b}||$.

7. Given the points $A\ (1, -2, 7)$ and $P\ (x, y, z)$ and the vector $\underline{n} =< 1, -5, 3 >$:

   (a) Write an equation that says the following: $\underline{n}$ and $\vec{AP}$ are perpendicular.

(b) What is the geometric description of the set of points $P\ (x, y, z)$ which satisfy the equation?

8.  A parallelepiped is a 3-d object with parallelograms for side, like a tilted box. Its volume is the area of its base times its height (in the direction perpendicular to that base). The three vectors $\underline{a} = 6\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\underline{b} = \mathbf{i} + 5\mathbf{j}$, $\underline{c} = 2\mathbf{i} + 2\mathbf{j} + 7\mathbf{i}$ determine a parallelepiped. Find its volume. Find a formula for the volume of a parallelepiped with sides $\underline{x}, \underline{y}, \underline{z}$.