Merit Worksheet 5 - Math 242, Fall 2005

1. (a) Find the parametric and symmetric equations of the line through the points $P(-1, 0, 2)$ and $Q(2, 1, 1)$.

   (b) Find the midpoint of PQ and check that it satisfies the equation of the line.

2. Find an equation for the plane through the points $P(7, 2, 1)$, $Q(6, -1, -3)$ and $R(9, 3, 2)$.

3. Find the equation of the plane through $P(1, -1, 1)$ that intersects the xy-plane in the same line as does the plane $3x + 2y - z = 6$.

4. Find the angle θ between the planes with equations

$$3x - 12y + 4z = 12 \quad \text{and} \quad 7x - 4y - z = 11$$

   Write symmetric equations for their line of intersection L.

5. Let the plane H be given by the equation $x + y + z = 10$. Write down the equation of a sphere of radius 2 which is tangent to the plane at some point.

6. Find the equations of the planes that are parallel to the plane $x + 2y - 2z = 1$ and two units away from it.

7. What cases can arise when you have two lines L_1 and L_2 in the xy-plane? What does this tell us about two linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ in two unknowns x and y?

8. What are the possible configurations for

   (a) Three lines in $\mathbb{R}^3$?

   (b) Three planes in $\mathbb{R}^3$?

   What does this tell you about solutions of suitable systems of linear equations?

Warm-Up Problems for Tuesday

1. Show that the graph of the curve with parametric equations $x = t\sin 6t, y = t\cos 6t, z = t$ lies on the cone $z = \sqrt{x^2 + y^2}$ with its vertex at the origin and opening upwards.

2. Suppose $\mathbf{r}(t) = 3\mathbf{i}\cos 2\pi t + 3\mathbf{j}\sin 2\pi t$. Find $\mathbf{r}'(7/4), \mathbf{r}''(7/4)$.