Merit Worksheet 7 - Math 242, Fall 2005

1. A particle has position vector $\mathbf{r}(t)$ and speed $v(t)$. Prove or disprove: Its scalar acceleration is $\frac{\mathrm{d}v}{\mathrm{d}t}$.

2. Which of the following best describes the path of a particle defined by the parametric equations

$$x(t) = \cos t^2, \qquad y(t) = \sin t^2?$$

   (a) a circle on which the particle moves faster and faster

   (b) a parabola on which the particle travels at constant speed

   (c) a parabola on which the particle travels faster and faster

   (d) a circle on which the particle moves slower and slower

   How could you parameterize the other options?

3. Which of the following is not a parametrization of the entire curve $y = x^3$?

   (a) $x(t) = t; \quad y(t) = t^3$

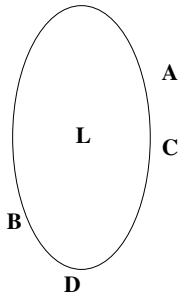
   (b) $x(t) = t^2; \quad y(t) = t^6$

   (c) $x(t) = t^3; \quad y(t) = t^9$

   (d) $x(t) = 2t; \quad y(t) = 8t^3$

   What are the differences between those that do parameterize the curve $y = x^3$?

4. A lighthouse at position L is in the middle of a lake. Its beam is turning counterclockwise with constant angular velocity. At which point



   (a) is the velocity vector of the beam largest?

   (b) is the velocity vector of the beam most parallel to $\mathbf{j}$?

(c) is the acceleration vector of the beam most parallel to $\mathbf{j}$?

5. Try to do this without looking at the book. Find the velocity, acceleration, speed and scalar acceleration of a moving point P whose trajectory is the helix with position vector

$$\mathbf{r}(t) = (a\cos\omega t)\mathbf{i} + (a\sin\omega t)\mathbf{j} + bt\mathbf{k}$$

This would be a typical trajectory for a charged particle in a constant magnetic field, which must satisfy Newton's Law $\mathbf{F} = m\mathbf{a}$ and the magnetic force law $\mathbf{F} = (q\mathbf{v}) \times \mathbf{B}$. (See, for example, p.796.) Suppose the constant magnetic field is vertical ($\mathbf{B} = B\mathbf{k}$). Find a relation between q, B, m, ω.

6. Try to do this without looking at the book. A projectile is launched from the point (x_0, y_0) at angle α with speed v_0. Assume it is travelling over flat ground, affected by gravity (32 ft/s^2) but not by air resistance.

   (a) What is its initial position vector?
   (b) What is its initial velocity vector?
   (c) What is its acceleration vector at time t?
   (d) What is its velocity vector at time t?
   (e) What is its position vector at time t?

7. A baseball is hit with an initial velocity of 96 ft/s an initial inclination angle of 15° from ground level straight down a foul line. Because of spin it experiences a horizontal acceleration of 2 ft/s^2 perpendicular to the foul line; otherwise air has no effect on its motion. (Obviously gravity affects its motion.) When the ball hits the ground, how far is it from the foul line?

8. Let $\mathbf{f} : \mathbb{R} \to \mathbb{R}^3$. Give a definition of $\lim_{t\to a} \mathbf{f}(t)$ that does not involve the component functions of $\mathbf{f}$. Show this agrees with your componentwise definition? Can you do the same for differentiation and integration?

Problems 2-4 are (slightly modified) from the Conceptests of Mark D. Schlatter, Dept. of Mathematics, Centenary College of Louisiana.

Warm-Up Problem for Thursday:

What are arc-length and curvature? Find the curvature κ of the plane curve $y = \cos x$ at $(0, 1)$.