Merit Worksheet 8 - Math 242, Fall 2005

1. Suppose that a point moves along the surface of a sphere. Show that its position vector is always perpendicular to its velocity vector.

2. Give an example of a curve in $\mathbb{R}^2$ or $\mathbb{R}^2$ for which the position at time $t = 10$ is the same as the position at time $t = 0$ but for which the speed is never 0.

   Can you find a differentiable function of a single variable with this property, i.e., a function $y = f(x)$ for which $f(10) = f(0)$ and $f'(x) \neq 0$ for $x \in [0, 10]$? Why or why not?

3. Consider the curve $\mathbf{r}(t) = \sin t\mathbf{i} + t\mathbf{j} + \cos t\mathbf{k}$, $0 \leq t \leq 2\pi$.

   (a) Find the arc length of $\mathbf{r}(t)$.

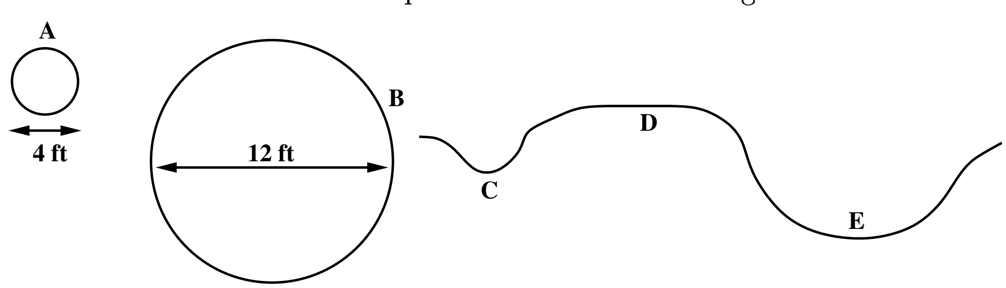
   (b) Find the arc length of $\mathbf{s}(t) = \sin t^2\mathbf{i} + t^2\mathbf{j} + \cos t^2\mathbf{k}$, $0 \leq t \leq \sqrt{2\pi}$.

   (c) Find the arc length of $\mathbf{w}(t) = \cos t\mathbf{i} + (t + \frac{\pi}{2})\mathbf{j} - \sin t\mathbf{k}$, $-\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$.

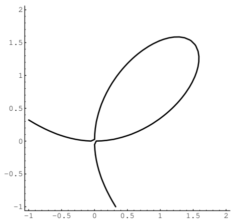
   (d) Is there a relationship between $\mathbf{r}, \mathbf{s}$ and $\mathbf{w}$?

   (e) What does this tell us about arc length?

4. What is the curvature at the points marked in the diagram?



5. Using the example of Problem 4, discuss the following terms: radius of curvature of a function, centre of curvature, osculating circle (or circle of curvature). Find these for the point $(3/2, 3/2)$ on the folium of Descartes, the curve defined by $x^3 + y^3 = 3xy$.



6. Using tangential and normal components of acceleration, describe those curves **r** for which

   (a) $\mathbf{a} \perp \mathbf{v}$.

   (b) $\mathbf{a}||\mathbf{v}$

      i. Assuming initial position $\mathbf{r}_0$ and initial velocity $\mathbf{v}_0$, find an equation for the velocity of $\mathbf{v}$.

      ii. Find an equation for the position $\mathbf{r}$.

      iii. Describe the line which contains the range of the position vector r.

7. Find the curvature, $\mathbf{T} = \hat{\mathbf{v}}$, N, the tangential and normal components of acceleration for the space curve with position vector $r(t) = < e^t \cos t, e^t \sin t, e^t >$.

Warm-Up for Thursday

Be prepared to hand these in.

1. In section 11.6, read pp. 827-830. (I will ask you to talk about this.)

   Given that the period of revolution of the earth around the sun is 365.26 days whereas that of Jupiter is 11.86 years, calculate the major semiaxis (in astronomical units) of the orbit of Jupiter.

2. Section 11.7, Q.32.