

Merit Worksheet 9 - Math 242, Fall 2005

1. Assume the earth has radius 3960 miles, period of revolution 365.26 days, and that the major semiaxis of its orbit is 92,956,000 miles. The moon period of revolution 27.32 days about the earth and the major semiaxis of its orbit is 238,900 miles.
 - (a) Suppose that an earth satellite in elliptical orbit varies in altitude from 100 to 1000 miles above the earth's surface (assumed spherical). Find this satellite's period of revolution.
 - (b) Find the altitude above the earth's surface of a satellite in a circular orbit that has a period of revolution of 1 hour. Anything odd?
2. Sketch or describe the traces of $x^2 - y^2 + z^2 = 1$ in both horizontal and vertical planes parallel to the coordinate axes.
3. **True/False:** The equation for the cylinder through the circle $x^2 + y^2 = 4$ is $z = x^2 + y^2 - 4$. What are the horizontal traces and the rulings? What are their equations?
4. Show that the projection into the xy -plane of the intersection of the plane $z = y$ and the paraboloid $z = x^2 + y^2$ is a circle.
5. Write an equation for the surface generated by rotating the curve $y^2 + z^2 = 1$ about the z -axis. What are its traces parallel to the coordinate planes?
6. By using substitutions like $x = X + h, y = Y + k, z = Z + l$ in the following equations, simplify them and recognize that their graphs are quadrics. Name the quadric and sketch it.
 - (a) $x^2 + y^2 + z^2 - 12x - 6y - 4z = 0$
 - (b) $z = x^2 + y^2 + 2x - 2y$
 - (c) $y^2 + 2(z - 1)^2 = (x - 2)^2$
7. Show that the following are equations of surfaces of revolution. Describe the axis of revolution and the generating curve.
 - (a) $z - 5 = (x - 2)^2 + (y + 3)^2$

(b) $y = -x^2 - z^2 + 2x - 4z - 1$

8. You can figure out the next two without the book.

If you know the cartesian coordinates of a point (x_0, y_0, z_0) , how do you describe its position in terms of r, θ, z ? Here r is the distance from the origin to $(x_0, y_0, 0)$ and θ is the angle between $(x_0, y_0, 0)$ and the x -axis.

Why are these called cylindrical coordinates? How do you move from cylindrical coordinates to cartesian coordinates?

What are the cylindrical coordinates of a sphere?

9. If you know the cartesian coordinates of a point (x_0, y_0, z_0) , how do you describe its position in terms of ρ, ϕ, θ ? Here ρ, ϕ, θ are as in the diagram. Why are these called spherical coordinates? How do you convert from spherical coordinates to cartesian coordinates?

Find the spherical coordinates of the point $(4, -3, 12)$.

Problems for next Tuesday

Come prepared! You should already know the material covered in lectures before you come to the Merit room. Hand these in on Tuesday.

1. What is the domain of $f(x, y, z) = \frac{1}{x^2-1} + \ln \frac{1}{4-y} + \frac{\tan z}{\sqrt{z-2}}$?
2. Sketch the graphs of the equations: $\theta = 3\pi/4, r = 4 \cos \theta$.