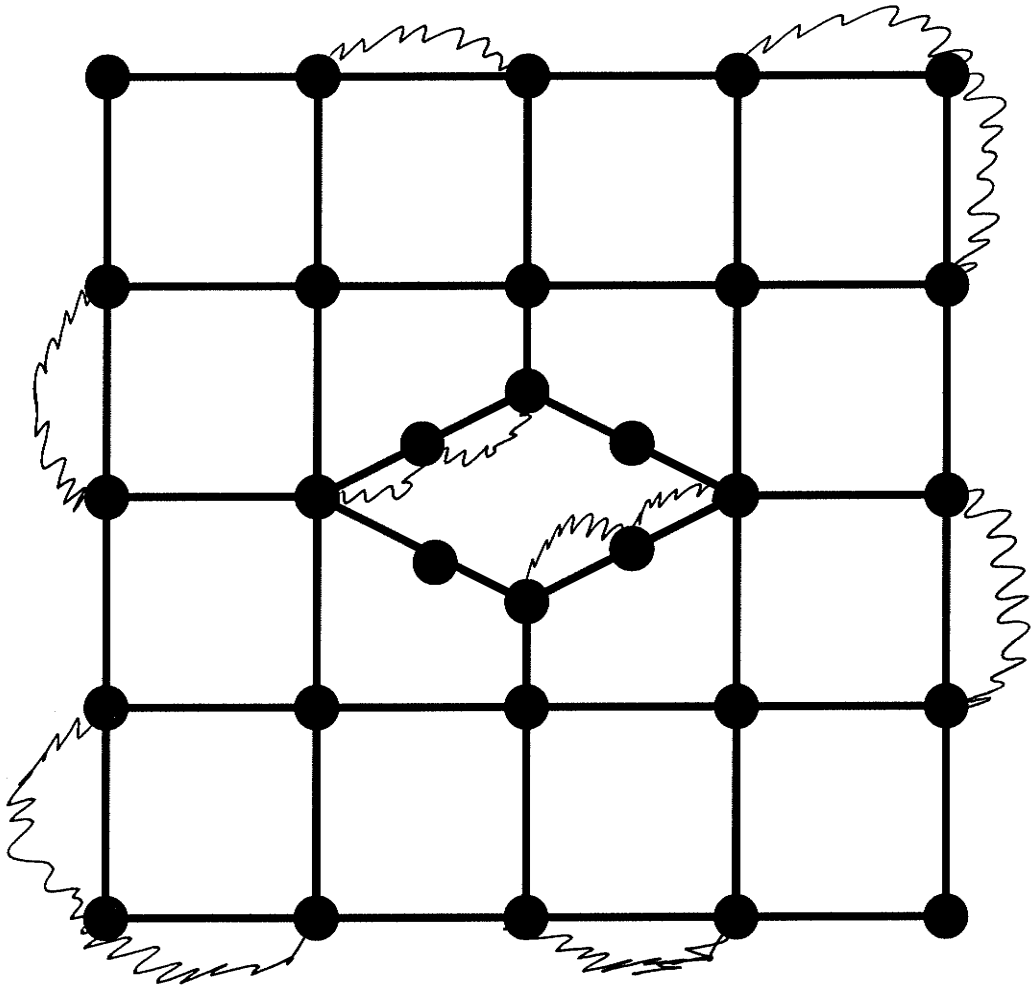
Math 181: Exam 1

Fall 2008, Tom Cooney

Name: Answer Key	
Problem 1:	/10
Problem 2:	/10
Problem 3:	/10
Problem 4:	/10
Problem 5:	/12
Problem 6:	/~~10~~ 8
Problem 7:	/~~10~~ ~~12~~ 13
Problem 8:	/14
Problem 9:	/~~12~~ 13
Total:	/100

You have fifty minutes to complete this test. Answer *all* questions.
Calculators and electronic devices are not allowed.

1. (10 Points) Find an efficient eulerization of the following graph.

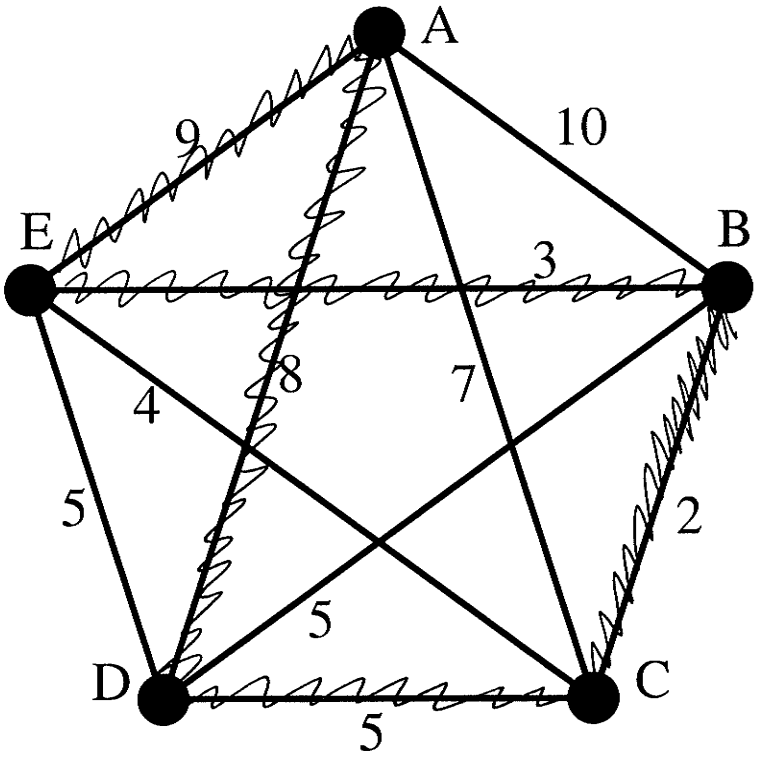


12 duplicated edges.

Edgewalker gives efficient eulerization for rectangular graphs, suggesting how to eulerize the exterior. Interior: Pair up vertices of odd valence and duplicate the edges on the path between them.

All vertices should have even valence at end (so that an Euler circuit then exists)
Duplicate existing edges, rather than creating brand new ones.

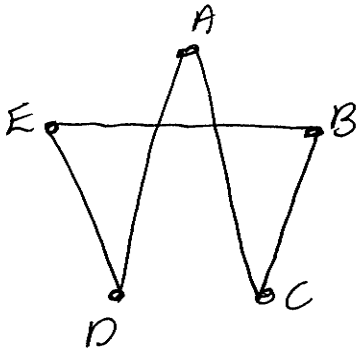
2. (10 Points) Apply the Sorted Edges Method to the graph given below. Describe your answer by listing the order in which it passes through vertices.



10
9
8
7
5
5 } One of these
5
4 x
3
2

ADCBEA

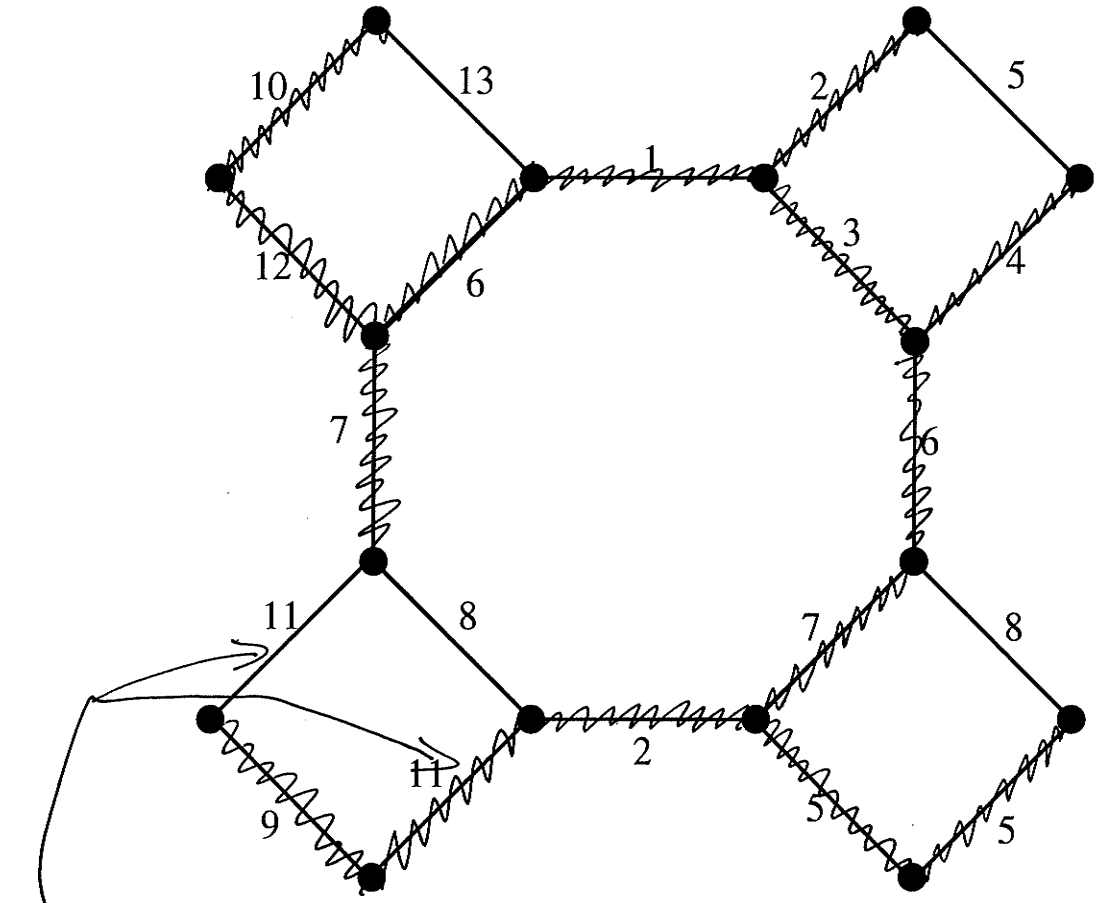
or



ACBEDA

Answer should be a Hamiltonian Circuit.

3. (10 Points) Apply Kruskal's algorithm to the graph below. Make sure to clearly indicate what edges are part of your answer.



One of these two edges.

Spanning Tree - but not Kruskal's Algorithm -7 (2 or 2 per error)

Basically Kruskal but with errors (-1) per edge

Circuits ~~(-3)~~ (But extra deductions for other errors still possible)

Need all ~~edges~~ vertices (-3) "

Not Connected (-3) "

(with more deductions for more unconnected components)

4. (10 Points) You are eulerizing a graph which has a total of 10 vertices. 6 of these vertices have odd valence.

   (a) What is the minimum number of edges you need to duplicate in order to eulerize the graph? Explain your answer.

3 edges.
Duplicating 1 edge can alter the valence of at most 2 vertices.
6 vertices — require at least 3 edges duplicated.
Depending on graph, may require more.

   (b) After eulerizing a graph, we often "squeeze" our answer back onto the original graph. Why?

In a real-world application, this would correspond to reusing an existing road, rather than building a new road.

(+2 for because the original graph is the one we are interested in)

5. (12 Points) Which of the following options best describes the situations given below?

- The chinese postman problem.
- The travelling salesman problem.
- Finding a minimal cost spanning tree.
- The machine-scheduling problem.
- The bin-packing problem

(a) A company links its offices with high-speed internet cables.

Minimal Cost Spanning Tree

(b) A pizza delivery person takes pizza to ten houses and then returns to the pizzeria.

Travelling Salesman Problem

(c) A postal worker picks up mail from six collection boxes around the city.

Travelling salesman Problem

(d) A furniture-maker requires pieces of timber of varying length and is buying ten-foot long boards of timber at the hardware store.

Bin-Packing Problem

(e) Five friends are preparing Thanksgiving dinner together.

Machine-Scheduling Problem

(f) An inspector examines the condition of the roads in a small town.

Chinese Postman Problem

6. (~~10~~ 8 Points) A thief is trying to log onto your online banking account. The thief knows that your password is:

- a mix of numerals (0-9) and lower-case letters (a-z).
- 7 symbols long
- starts with a letter
- does not repeat any symbols

How many potential passwords must the thief check?

$26 \times 35 \times 34 \times 33 \times 32 \times 31 \times 30$

26 letters — 26 letters, 10 numerals but not the already chosen letter.

7. (~~18~~ 13 Points)

(5) (a) Define the chromatic number of a graph.

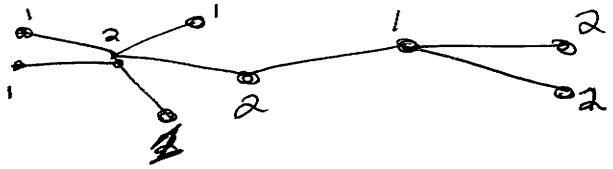
The minimum (+2) number of colours needed to colour (+1) each vertex of a graph so that no ~~two~~ adjacent vertices have (+2) the same colour.

(4) (b) What is the chromatic number of K_n? (2) Explain your answer. (2)

n. All n vertices are linked to all other vertices by edges and so must have different colours.

(4) (c) What is the chromatic number of a tree? (2) Explain your answer. (2)

2. As there are no circuits, we can alternately assign colours 1 and 2 to the vertices of the tree.

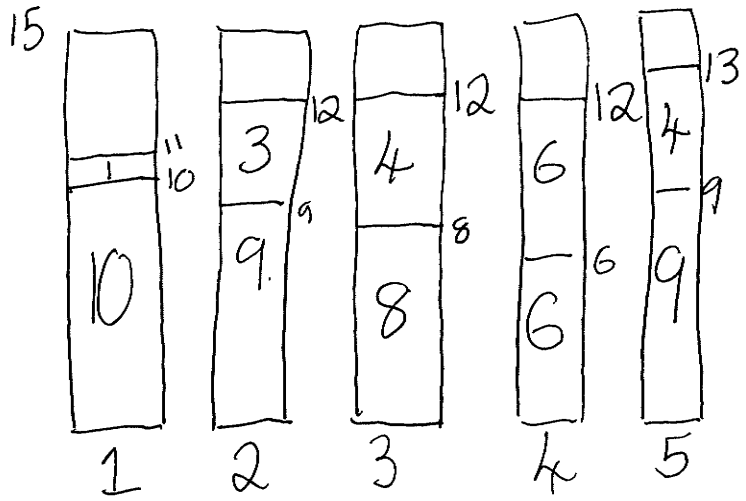


8. (14 Points) Alex has to back up all the computer files at Alex's workplace. Each DVD can hold at most 15 gigabytes. The files to be backed up are of size

$$10,\ 9,\ 8,\ 4,\ 3,\ 6,\ 6,\ 1, 9,\ 4.$$

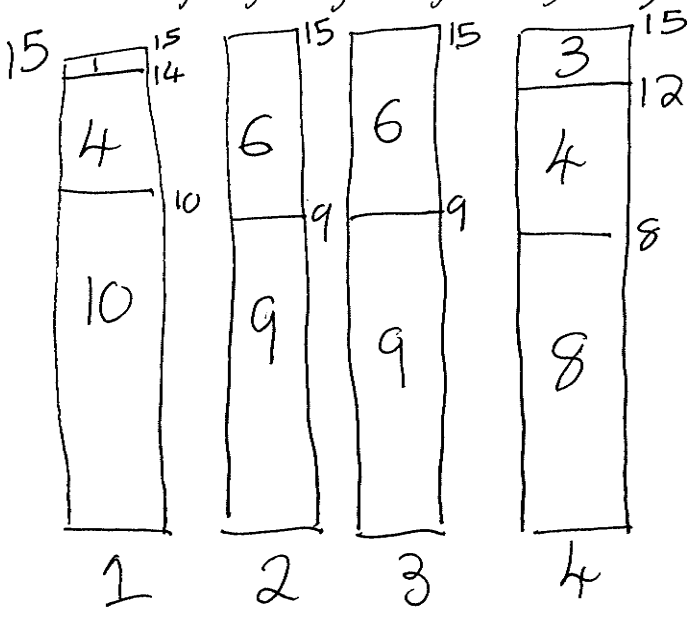
How would the files be divided up:

(a) if Alex uses the worst fit (WF) algorithm?

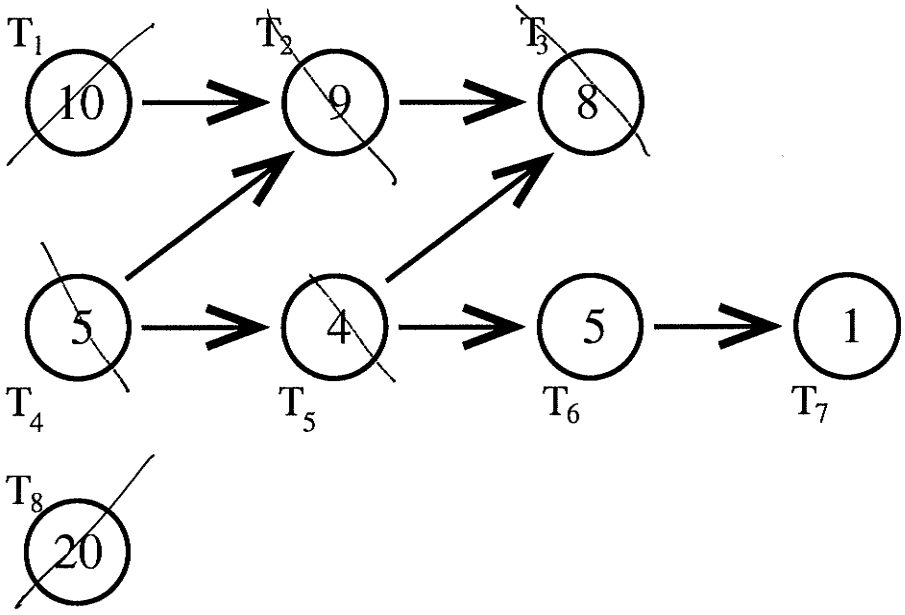


(b) if Alex uses the the first fit decreasing (FFD) algorithm?

10, 9, 9, 8, 6, 6, 4, 4, 3, 1



9. (~~12~~ 13 Points) Use the order-requirement digraph below to answer the following two questions. Note that you are *NOT* being asked to find a schedule.



5 (a) What priority list would you use if you apply the Critical Path Scheduling algorithm?

$T_1, T_4, T_8, T_2, T_5, T_3, T_6, T_7$

4 (b) Estimate the earliest possible completion time of these tasks if

i. there are two processors.

$$\frac{\text{Total Amount of Work}}{\text{Number of Processors}} = \frac{62}{2} = 31$$

4 ii. there are four processors.

Critical Path Length = 27

(T_1, T_2, T_3).