

Math 181 – Review for Exam 3 – Fair Division

1. Rich Aunt Gertrude dies and leaves her estate to her nieces and nephews. Aunt Gertrude being eccentric, they are not allowed to sell anything or to hire a professional valuer. Suppose that Aloysius, Bertha, and Cornelius each value the items below as follows (in hundreds of thousands of dollars):

Object	Aloysius	Bertha	Cornelius
Mansion	40	50	20
Miro Painting	40	20	30
Ferrari 250 GTO	20	30	50

Find a way of dividing out the goods that is:

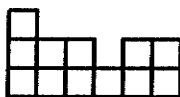
- (a) equitable
- (b) proportional
- (c) where someone envies someone else
- (d) envy-free
- (e) not Pareto-optimal
- (f) Pareto-optimal

Explain your answers.

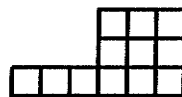
2. Use the adjusted winner procedure to resolve the labour dispute:

Issue	Labour	Management
Base Salary	30	40
Salary Increases	20	40
Benefits	35	5
Vacation time	15	5

3. Is divide-and-choose proportional? Envy-free? Use divide-and-choose to divide the following cake. (Let Archibald choose and Bonnie divide.)



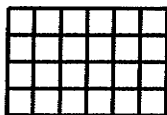
Archibald's view



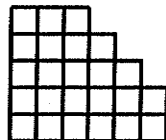
Bonnie's view

4. Would you rather divide or choose? If you know the other person's preferences? If you don't? Explain.
5. Use the lone-divider method to divide up the following cakes. (Let Agatha be the lone-divider.) Is the lone-divider method proportional? Envy-free? Explain.

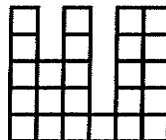
(a)



Agatha's view

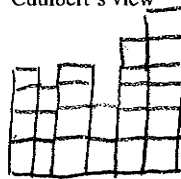
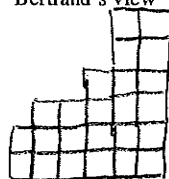
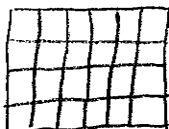


Bertrand's view



Cuthbert's view

(b)



Math 181 – Review for Exam 3 – Weighted Voting Systems

1. Consider the weighted voting system

$$[q : w_A, w_B, w_C] = [9 : 6, 4, 4].$$

- (a) Is there a dictator? A dummy voter? A voter with veto power?
- (b) List all the possible permutations. Who is pivotal in each permutation?
- (c) What is the Shapley-Shubik power index of this system?
- (d) List all the winning coalitions.
- (e) What is a critical voter? Who is critical in the above winning coalitions?
- (f) List all the blocking coalitions. Who is critical in the blocking coalitions?
- (g) What is the Banzhaf power index of this system?
- (h) Pair each winning coalition for which B is critical with its dual blocking coalition for which B is critical.

2. Consider the weighted voting system

$$[q : w_A, w_B, w_C, w_D, w_E, w_F] = [8 : 5, 1, 1, 1, 1, 1].$$

- (a) Describe the permutations in which A is pivotal.
- (b) Count the permutations in which A is pivotal.
- (c) Find the Shapley-Shubik power index of this system.
- (d) Describe the winning coalitions in which A is critical.
- (e) Count the winning coalitions in which A is critical.
- (f) Describe the winning coalitions in which B is critical.
- (g) Count the winning coalitions in which B is critical.
- (h) Find the Banzhaf power index of this system.

3. Find the Shapley-Shubik power index of the weighted voting system

$$[q : w_A, w_B, w_C, w_D, w_E, w_F, w_G, w_H, w_I] = [7 : 3, 3, 1, 1, 1, 1, 1, 1, 1].$$

4. Find an example of a weighted voting system with four voters such that

- (a) there is a voter with veto power but no dummy voters.
- (b) all voters have equal power.
- (c) one voter has all the power.
- (d) there is no dictator and one dummy voter.
- (e) the winning coalitions are:

$$AB, AC, ABC, ABD, ACD, ABCD$$

- (f) Does the above have a dictator? A voter with veto power? A dummy?

Explain your answers.