# Math 220 AD9 Spring 2009 Worksheet 16

1. Differentiate the following functions

   $$(a)\quad y = (\cos x)^x \text{ for } |x| < \tfrac{\pi}{2} \quad (b)\quad y = x^{\sin x} \text{ for } x > 0 \quad (c)\quad y = x^{\sqrt{x}+\ln x} \text{ for } x > 0.$$

   (Hint: your first step should be to take the natural log of both sides of the equation.) Why are there restrictions on what $x$ we consider? What types of functions do you differentiate using logarithmic differentiation?

2. What is the difference between defining a function $y$ of $x$ explicitly and defining it implicitly? Give examples.
   What is the idea behind implicit differentiation? Is there anything important to remember? When do you use implicit differentiation?

3. Find the slope of the tangent line to the curve $x^3y - 4\sqrt{x} = x^2y$ at the point $(2, \sqrt{2})$. Do this first by solving for $y$ as a function of $x$ and then by using implicit differentiation.

4. Find the derivative $\frac{dy}{dx}$ (which can also be written $y'(x)$).

   $$(a)\quad e^{xy} = x^2 + xy \quad (b)\quad y = \cos(x+y) \quad (c)\quad \ln(x^2+y^2) = \frac{x+1}{y}$$

5. Find the second derivative $\dfrac{d^2y}{dx^2}$ (which can also be written $y''(x)$).

   $$(a)\quad x^{2/3} + y^{2/3} = 4 \quad (b)\quad xy = x + y^2$$

6. Suppose $y = \arctan x$. Write $x$ as a function of $y$. Use implicit differentiation to find $\frac{dy}{dx}$. Now write $\frac{dy}{dx}$ only in terms of $x$; no $y$-terms should appear.

7. Find the derivative of the following functions:

   $$(a)\quad \arcsin(x^3+1) \quad (b)\quad \frac{x^2}{\arctan x} \quad (c)\quad e^{\arccos\sqrt{x}} \quad (d)\quad \operatorname{arccot}(x^2+3)$$

8. Find the points on the curve $x = ye^y$ where the tangent line is horizontal or vertical.

9. Use a similar approach to (6) to find the derivative of $y = \operatorname{arcsec} x$.

10. Use implicit differentiation to show that the derivative of $\log x$ is $\frac{1}{x}$.

11. Find the derivative of $y = \sqrt{x+1}\sqrt[3]{x+2}\sqrt[4]{x+3}$. You can use the product and chain rules, but I think that you will find it much easier if you use logarithmic differentiation.

    **Note:** There is another trick to compute derivatives that you may find useful. Instead of taking the log of both sides, you can rewrite a function using exponentials and logs. For example, $f(x) = x^x = e^{\log x^x} = e^{x\log x}$. The derivative of the last form can be evaluated using rules you already know!

12. Find the derivative of $f(x) = x^x$ using the above method.
13. Find the derivative of $f(x) = x^{1/x}$ using the new method.

## Preparation for next time

Complete the following table:

| The function $f(x)$ | The derivative $f'(x)$ |
|---|---|
| $x^r$ | |
| $e^x$ | |
| $\ln x$ | |
| $\sin x$ | |
| $\cos x$ | |
| $\tan x$ | |
| $\csc x$ | |
| $\sec x$ | |
| $\cot x$ | |
| $\arcsin x$ | |
| $\arccos x$ | |
| $\arctan x$ | |
| $g(x)h(x)$ | |
| $\frac{g(x)}{h(x)}$ | |
| $g(h(x))$ | |