

## Math 220 AD9 Spring 2009 Worksheet 19

1. Use the definition of the derivative to find the derivative of  $y = x^2 + 2x + 1$ .
2. Differentiate the following functions:

$$(a) 7^x \quad (b) e^x \quad (c) x^e \quad (d) \pi^e \quad (e) (x+1)^{(2x^2+1)}$$

3. Differentiate the following functions:

$$(a) (2x+3)\sqrt{x+5} \quad (b) x^2(x^3+1)(2x-1) \quad (c) \frac{x^4 - \sqrt[3]{x+1}}{\ln x} \quad (d) e^{\sin(x^3+1)}$$

4. Consider the functions:

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}.$$

Show that the derivative of  $\sinh x$  is  $\cosh x$  and vice versa.

5. Using the Mean Value Theorem, show that for all  $x$  and  $y$ ,

$$\cos x - \cos y \leq |x - y|.$$

6. Find the equation of the tangent line to the curve  $4x^2 + 9y^2 = 25$  at the point  $(2, -1)$ . Find all the points where the tangent lines are either horizontal or vertical.
7. If  $f$  and  $g$  are differentiable functions on the interval  $[a, b]$  with  $f(a) = g(a)$  and  $f(b) = g(b)$ , prove that at some point in the interval  $[a, b]$ ,  $f$  and  $g$  have parallel tangent lines.
8. Differentiate the function  $f(x) = \arctan \frac{x}{x+1}$  - this could also be written  $\tan^{-1} \frac{x}{x+1}$ .
9. Find all function  $g(x)$  such that  $g'(x) = \sqrt{x}$ .

10. Find an equation for the tangent line to the curve

$$y^2 + xe^y = 4 - x,$$

at the point  $(2, 0)$ .

11. The height of a ball at time  $t$  is given (in feet) by

$$s(t) = -16t^2 + 64t + 32.$$

At what speed was the ball initially thrown into the air? From where? What is the greatest height it reaches?

12. Find  $f^{(34)}(x)$  for  $f(x) = \cos 2x$ .

**Good luck in the exam!**