

## Math 220 AD9 Spring 2009 Worksheet 29

1. Differentiate the following functions:

$$x^3 + 5, \quad x^3 - 7, \quad x^3 + 1, \quad x^3 - 13.$$

The function  $F(x)$  is an *antiderivative* of  $f(x)$  if  $F'(x) = f(x)$ .

Find an antiderivative of  $3x^2$ . Find a different antiderivative of  $3x^2$ . Describe *all* the antiderivatives of  $3x^2$ . How do you know that this is all of them? (We discussed this earlier this semester.) Sketch graphs of some antiderivatives of  $3x^2$ . What is the relation between the graphs of all the antiderivatives of a function  $f(x)$ ? Explain.

2. Find an antiderivative of  $8x^3 + 9x^2 + 2$ . Find another antiderivative of this function. Find all the antiderivatives of this function.
3. We find an antiderivative by “reversing” or “undoing” differentiation. So  $x^6$  and  $x^6 + 1$  are both antiderivatives of  $6x^5$ . All of the antiderivatives of  $6x^5$  are of the form  $x^6 + c$ , where  $c$  is any constant.

We call  $x^6 + c$  the *indefinite integral* of  $6x^5$  – it is indefinite because  $c$  could be any real number. If  $F(x)$  is the antiderivative of  $f(x)$ , then

$$\int f(x) dx = F(x) + c,$$

is the *indefinite integral* of  $f(x)$ , where  $c$  is an arbitrary constant (called the *constant of integration*).

Yes, include the  $dx$  every time you write down an integral – we will talk more about what the  $dx$  means later.

Always include the constant of integration in your answer.

What is the indefinite integral of  $\sqrt{x}$ ? What is  $\int x^3 + 2x - 7 dx$ ?

4. We can see that any two antiderivates of the same function must differ only by a constant. If  $F_1$  and  $F_2$  are both antiderivatives of  $f$ , then  $F_1'(x) - F_2'(x) = f(x) - f(x) = 0$ , so  $F_1$  and  $F_2$  have the same derivatives, which shows that they can only differ by a constant. Consider the following functions:

$$F_1(x) = \frac{1}{1-x} \quad \text{and} \quad F_2(x) = \frac{x}{1-x}.$$

Show that these are both antiderivatives of  $f(x) = 1/(1-x)^2$ . Explain why this is not a contradiction.

5. What is the power rule for derivatives? What is the power rule for integration, i.e., what is  $\int x^n dx$ ?

What is  $\int 3x^7 + x^4 - \sqrt[3]{x} + x^{0.1} - 3 + \frac{1}{x} + \frac{1}{x^3} dx$ ?

6. Joe throws a ball straight upward at an initial speed of  $40\text{ ft/s}$ . Gravitational acceleration is  $32\text{ ft/s}^2$  downwards. Find a function that describes the position of the ball at each time  $t$ .

7. Evaluate the following:

$$\int \sin x \, dx, \quad \int \cos x \, dx, \quad \int e^x \, dx, \quad \int \ln |x| \, dx$$

Check your answers by differentiating.

8. Find a function  $f(x)$  satisfying the following conditions:  $f'(x) = \sin x + 1$ ,  $f(\pi/2) = -3$ .

9. Find all the functions  $f(x)$  satisfying  $f'''(x) = \sqrt{x} + 2 \cos x$ .

10. Evaluate the following:

$$\int x^{1/4} (x^{2/3} - x^{-1/2}) \, dx, \quad \int \frac{(e^x)^4 - 5}{e^x} \, dx.$$

11. Determine the position function if the acceleration function is  $a(t) = 3 \sin t$ , the initial velocity is  $-2\text{ ft/s}$ , and the initial position is  $s(0) = 4$ .

12. Evaluate the following:

$$\int \frac{1}{\sqrt{1-x^2}} \, dx, \quad \int \frac{1}{1+x^2} \, dx, \quad \int \frac{x^2+1}{x} \, dx, \quad \int \frac{1}{\sqrt[3]{2}} \, dx$$

13. What is the derivative of  $f(x) = \sin(e^x)$ ?

Then what is  $\int \cos(e^x) e^x dx$ ? (Don't forget  $+C!!!$ )

14. What is the derivative of  $f(x) = (x^3 + x^2 + 1)^{10}$ ?

Then what is  $\int 10(x^3 + x^2 + 1)^9 (3x^2 + 2x) \, dx$ ? (Don't forget  $+C!!!$ )

## Preparation for next time

For Monday, read section 4.2. There will be a preparation quiz.