

### Math 220 AD9 Spring 2009 Worksheet 3

1. Without using your calculator, find the exact values of:

(a)  $\ln(e^{\sqrt{2}})$

(b)  $\log_6\left(\frac{1}{36}\right)$

2. Solve each equation for  $x$ :

(a)  $\ln(2x - 1) = 3$

(b)  $e^{e^x} = 4$

(c)  $\log_x 32 = 5$

(d)  $\ln(x^2 - \pi^2) - \ln(x + \pi) = 7$  (also find the domain of  $\ln(x^2 - \pi^2) - \ln(x + \pi)$ .)

3. The Richter magnitude  $M$  of an earthquake is defined in terms of the energy  $E$  in Joules released by the earthquake, with  $\log_1 0E = 4.4 + 1.5M$ . Find the energy for earthquakes with magnitudes 4, 5, and 6, respectively. For each increase in  $M$  of 1, by what factor does  $E$  change?

4. Without using your calculator, estimate  $\log_4 \frac{1}{10}$ ,  $\log_4 60$ ,  $\log_3 7$ .

5. Simplify

$$\frac{2^3 \cdot 7^{3/2} \cdot 5^{1/2}}{2^5 \cdot \sqrt{175} \cdot 8^{1/3}}$$

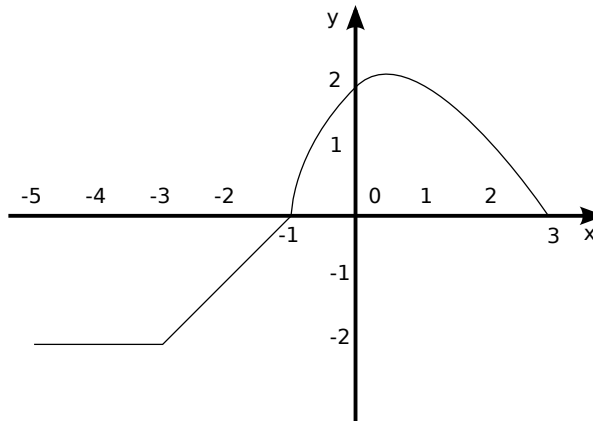
6. Let  $g(z) = z^2 + 1$  and  $h(z) = 2z - 1$ . Find the compositions  $(g \circ h)(z)$  and  $(h \circ g)(z)$ .

7. Let  $f(x) = x^3$  and  $g(x) = \cos x$ . Find the compositions  $(f \circ g)(x)$  and  $(g \circ f)(x)$ . Are the composite functions equal? Why or why not?

8. Let  $f(x) = x^2$  and  $g(x) = \sqrt{x}$ . Find the composites  $(f \circ g)(x)$  and  $(g \circ f)(x)$ . Are these composite functions equal?

9. Let  $h(x) = (x^2 + 4)^{3/2}$ . Find functions  $f$  and  $g$  such that  $h(x) = f(g(x))$ . Is there more than one possibility for  $f$  and  $g$ ?

10. Let  $f(x)$  be the function graphed below. First explain in words how to obtain the graphs of the functions listed below and then draw rough sketches of these graphs.



- |               |                        |
|---------------|------------------------|
| (a) $-f(x)$   | (b) $f(-x)$            |
| (c) $2f(x)$   | (d) $f(2x)$            |
| (e) $f(x+3)$  | (f) $f(x)+3$           |
| (g) $f(2x-6)$ | (h) $f(\frac{x}{2})-6$ |

- Explain in words how to obtain the graph of  $y = 2x^2 - 4x + 3$  from the graph of  $y = x^2$ .
- For each of the following, find a function of the form  $y = Ca^x$  that passes through the supplied points.
  - $(0, 2)$  and  $(1, 4)$ .
  - $(-1, 2)$  and  $(0, 1)$ .
  - $(-2, 0.0625)$  and  $(2, 16)$ .
- If you had two functions  $a^x$  and  $b^x$ ,  $a$  and  $b$  both positive, what information would you need to determine which was larger for *negative* values of  $x$ ? How little information about  $a$  and  $b$  can we get away with and still be able to answer the question?
- Sketch a graph of the function  $f(x) = 2^x$ , labeling a few points. For positive  $x$  values, which function seems to be increasing the most quickly:  $2^x$ ,  $x^2$ , or  $x^2 0$ ?
- Sketch graphs of  $2^x$ ,  $5^x$ ,  $1^x$ , and  $(1/2)^x$  on the same axes. Label the y-intercepts. For positive  $x$ , which is largest? Which functions are increasing everywhere? Which functions are decreasing everywhere?
- It's known that for all positive numbers  $a$ ,  $0^a = 0$ . It's also true that for all positive numbers  $b$ ,  $b^0 = 1$ . Given those two facts, what value do you think  $0^0$  should take? What does the graph of  $x^x$  look like near zero?
- Find all possible solutions of the following equations.
  - $\sin^2 x + 2 \sin x = -1$ .
  - $\cos^3 x - \cos x = 0$ .
  - $4 \sin^2 x + 2(1 - \sqrt{3}) \sin x = \sqrt{3}$ .

18. Below are a number of different trigonometric expressions involving or related to  $\sin x$ . Identify which expressions are equal to each other. (Don't expect them to all pair off!)

- (a)  $\sin^2 x$       (b)  $\sin(x^2)$       (c)  $\csc x$       (d)  $(\sin x)^2$       (e)  $\sin^{-1} x$   
(f)  $2 \sin x$       (g)  $(\sin x)^{-1}$       (h)  $\arcsin x$       (i)  $\frac{1}{\sin x}$       (j)  $\sin 2x$   
(k)  $(\csc x^2)^{-1}$       (l)  $\frac{\sin 2x}{\cos x}$       (m)  $1 - \cos^2 x$       (n)  $\cos^2 x - \cos 2x$

19. What are the zeros and asymptotes of

$$\frac{x^2 - 4}{x^4 - 9x^2} ? \quad \frac{x - 4}{x^2 - 3x - 4} ?$$

20. (a) Sketch two functions which both have domain  $[0, 20]$  and range  $[-10, 10]$  but which do not intersect.

(b) Sketch a function which has domain  $[-10, 10)$  and range  $[0, \infty)$ .

21. What is  $\arcsin(\sin x)$ ? Are you sure about that?

## Preparation for next time

Read Section 1.2. Draw the graph of a function  $f(x)$  such that

$$\lim_{x \rightarrow 2} f(x) = 4, \quad f(2) = 0.$$