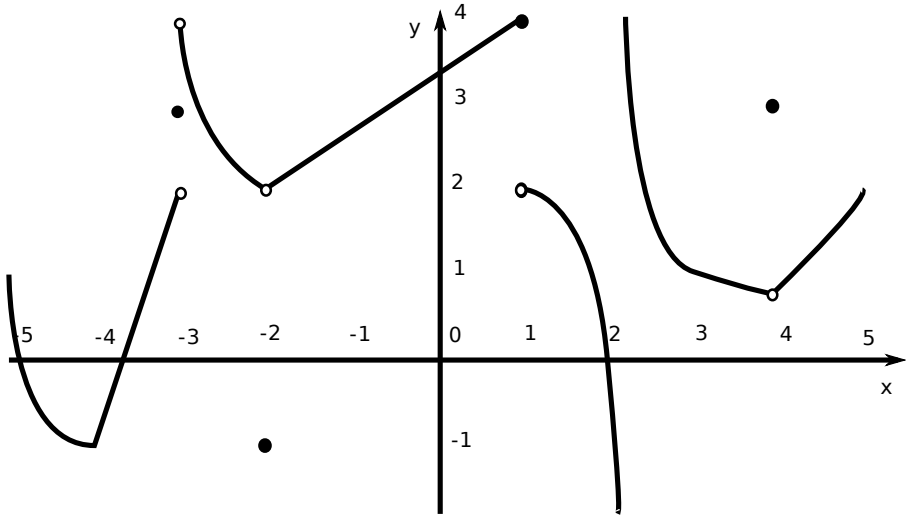
Math 220 AD9 Spring 2009 Worksheet 4

1. True or False: As x increases to 100, $f(x) = 1/x$ gets closer and closer to 0, so the limit as x goes to 100 of $f(x)$ is 0. Be prepared to justify your answer.

2. Multiple Choice: The statement "Whether or not $\lim_{x\to a} f(x)$ exists depends on the value of f(a)" is true...
   (a) always (b) sometimes (c) never
   Draw some examples to back up your answer.

3. The graph of $y = f(x)$ is shown below. Evaluate the following limits:

$$\begin{array}{llllllll} (a) & \lim\limits_{x\to -4^-} f(x) & (b) & \lim\limits_{x\to -4^+} f(x) & (c) & \lim\limits_{x\to -3^-} f(x) & (d) & \lim\limits_{x\to -3^+} f(x) \\ (e) & \lim\limits_{x\to -2} f(x) & (f) & \lim\limits_{x\to 1} f(x) & (g) & \lim\limits_{x\to 2^+} f(x) & (h) & \lim\limits_{x\to 2^-} f(x) \\ (i) & \lim\limits_{x\to 4^-} f(x) & (j) & \lim\limits_{x\to 4^+} f(x) & (k) & f(-3) & (l) & f(1) \end{array}$$



4. Use your calculate to estimate the following limits:

$$\lim_{x\to 1} \frac{x^3-1}{x^2+x-2}, \qquad \lim_{x\to 2} \frac{x}{x-1}, \qquad \lim_{x\to\infty} \frac{100x^2+500}{x^3}, \qquad \lim_{x\to 4} \frac{x^3-3x^2+4x}{x^2-16}.$$

   Can you explain what value each of these limits takes and why?

5. Consider

$$\lim_{x\to 0} \cos\frac{1}{x}.$$

   What values does this function take at $x = \frac{1}{2\pi}, \frac{1}{4\pi}, \frac{1}{6\pi}, \ldots$? At $\frac{2}{\pi}, \frac{2}{3\pi}, \frac{2}{5\pi}, \ldots$? What is the value of this limit? Why?

   Now consider $\lim\limits_{x\to 0} x\cos\frac{1}{x}$. What is the value of this limit? Why?

6. Play lawyer and decide which of the following is the best description of what is meant by the mathematical statement

$$\lim_{x \to 2} f(x) = 5$$

   (a) If you give me a distance d, however small, I can show you a z close to 2 so that $f(z)$ is closer than d to 5.

   (b) $f(2) = 5$

   (c) If you give me a distance d, however small, I can give you a distance e so that if you take *any* number, z, closer to 2 than e then $f(z)$ will be closer to 5 than your d.

7. Consider the functions $f(x) = \frac{|x|}{x}$ and $g(x) = \frac{x^2-4}{x-2}$

   (a) Write f as a piecewise-defined function, and find its domain and range.

   (b) Simplify g and find its domain and range.

   (c) Sketch graphs of f and g.

   (d) If you did the above correctly, each of f and g has a hole in its graph. If you wanted to "fill" this hole in the graph of g, what y-value would you choose? Can we do this for f?

8. **Zeno's Paradox**

   Assume that you are walking across the quad. First you walk half way, get tired and stop. Then you walk half of the remaining distance, get tired and stop again. Then you again walk half the distance remaining, see someone cute and stop to check them out. Then you again walk half the distance, get hit in the head by a Frisbee and get knocked out. You come to, and continue to be interrupted every time you go half of the remaining distance.

   (a) If the total distance across the quad is $1q$ (q is for the distance unit "quadlength"), write down a sequence $\{a_n\}$ for how long the n^{th} leg of your journey is.

   (b) Write down another sequence $\{b_n\}$ for how far you have traveled after the n^{th} leg of your journey.

   (c) What is the connection between the two sequences you have generated?

   (d) Do you ever reach the other side of the quad?

Preparation for next time

Read Section 1.3. Use the rules for limits to find:

$$\lim_{x \to 2} \frac{x+2}{x^3 - 9} + 3x.$$

9. For the following find a function of the form $f(t) = t^k$, $k \neq 1$, and a function g such that $(f \circ g)(x) = h(x)$.

   (a) $h(x) = (x+1)^5$

   (b) $h(x) = \frac{1}{(x^4+x^3+1)^6}$

   (c) $h(x) = \sqrt[4]{x^2-9}$

   (d) $h(x) = \frac{1}{\sqrt[5]{(x^3-2)^3}}$

   (e) $h(x) = \frac{1}{x+1}$

   (f) $h(x) = (1+x^4)^{17}$

   (g) $h(x) = \sin^2(2x+1)$

   Why did I specify $k \neq 1$?

10. Evaluate the following limits if they exist, or explain why they do not exist. Use algebraic methods or sketch a graph by hand.

    (a) $\lim_{x\to 5} \dfrac{5x-25}{x-5}$.

    (b) $\lim_{x\to 4^-} \dfrac{1}{x-4}$.

    (c) $\lim_{x\to 8} \dfrac{1}{x-8}$.

    (d) $\lim_{x\to -3} \dfrac{x}{x-3}$.

    (e) $\lim_{x\to 0} \dfrac{|x|}{x}$.

    (f) $\lim_{x\to 0} \dfrac{|x|}{x^2}$.