Math 220 AD9 Spring 2009 Worksheet 42

1. What is the relationship between the functions for position, velocity, and acceleration of a particle at time t? If you know one of these, how can you find the others? If I tell you the acceleration, what else do you need to know to find the position at time t?

2. Roadrunner races off a cliff, over a deep canyon, and runs safely to the other side. Wile E. Coyote chases him but makes the error of looking down while halfway across the canyon. He then falls. Eight seconds pass before he hits the ground. How deep is the canyon? What speed is he going when he hits the ground?

3. A cannonball has been fired at an angle of $60°$ from the horizontal at an initial speed of $240 ft/s^2$. We will keep track of its location using two separate functions. Its left-to-right position at time t is $x(t)$ and its height above the ground at time t is $y(t)$.

   (a) What are $x(0)$ and $y(0)$?

   (b) What are $x'(0)$ and $y'(0)$?

   (c) What are $x''(t)$ and $y''(t)$?

   (d) What are $x'(t)$ and $y'(t)$?

   (e) What are $x(t)$ and $y(t)$?

   (f) When does the cannonball hit the ground again? (Assume all of this is taking place on a flat stretch of land.) (Time of flight)

   (g) Where does the cannonball hit the ground? (Horizontal range)

   (h) What is the greatest height that the cannonball reaches?

4. You can measure your reaction time using a ruler. Hold your thumb and forefinger on either side of a ruler. Have someone in your group drop the ruler. You grab it as fast as you can. Take the distance d that the ruler falls and compute how long the ruler fell. What is the relationship between d and your reaction time?

   For comparison purposes, a top athlete has a reaction time of about 0.15 seconds. Who's the fastest at your table? No cheating.

5. Astronaut Alan Shepard played a few golf shots on the moon. The gravitational force on the moon is roughly one-sixth that of earth. (Use $g = 5.2\ ft/s^2$.) Would a golf ball would travel six times as high and six times farther on the moon than on Earth? Assume that the ball is hit at initial speed $60 ft/s$ at any angle of $30°$ above the horizontal.

6. A plane at a height of 256 feet wants to drop medical supplies to a specific location on the ground. If the plane has a horizontal velocity of $100 ft/s$, how far away from the target should the plane release the supplies in order to hit the target location?

7. Evaluate the following limits:

   (a) $\lim_{x\to\infty} \dfrac{x^3+3x+1}{x^2+1}$

   (b) $\lim_{x\to-\infty} \dfrac{x+100}{x^2+x+3}$

   (c) $\lim_{x\to\pi/2} \dfrac{x}{\sin x}$

   (d) $\lim_{x\to 0} \dfrac{2x}{\sin 3x}$

   (e) $\lim_{x\to 2} \dfrac{x^2+x-6}{x-2}$

   (f) $\lim_{x\to 5} \dfrac{\ln x}{x}$

   (g) $\lim_{x\to 0} x \ln x$

   (h) $\lim_{x\to\infty} \dfrac{x^2+3}{e^{3x}}$

8. Use the definition of the derivative to find $f'(x)$ and $f'(2)$ for the function $f(x) = 3x^2 + 5x + 1$.

9. Write down a differentiation problem which requires using the product rule. Write down the product rule. Solve the problem you created.

10. Write down a differentiation problem which requires using the chain rule. Write down the chain rule. Solve the problem you created.

11. Write down a differentation problem which requires using the quotient rule. Write down the quotient rule. Solve the problem you created.

12. Write down a differentiation problem which requires using a combination of the differentiation rules. How do you know which rules to use and when? Solve the problem you created.

Preparation for next time

There is no next time!

In case I don't see you again, best of luck with all your exams. Have a great summer!