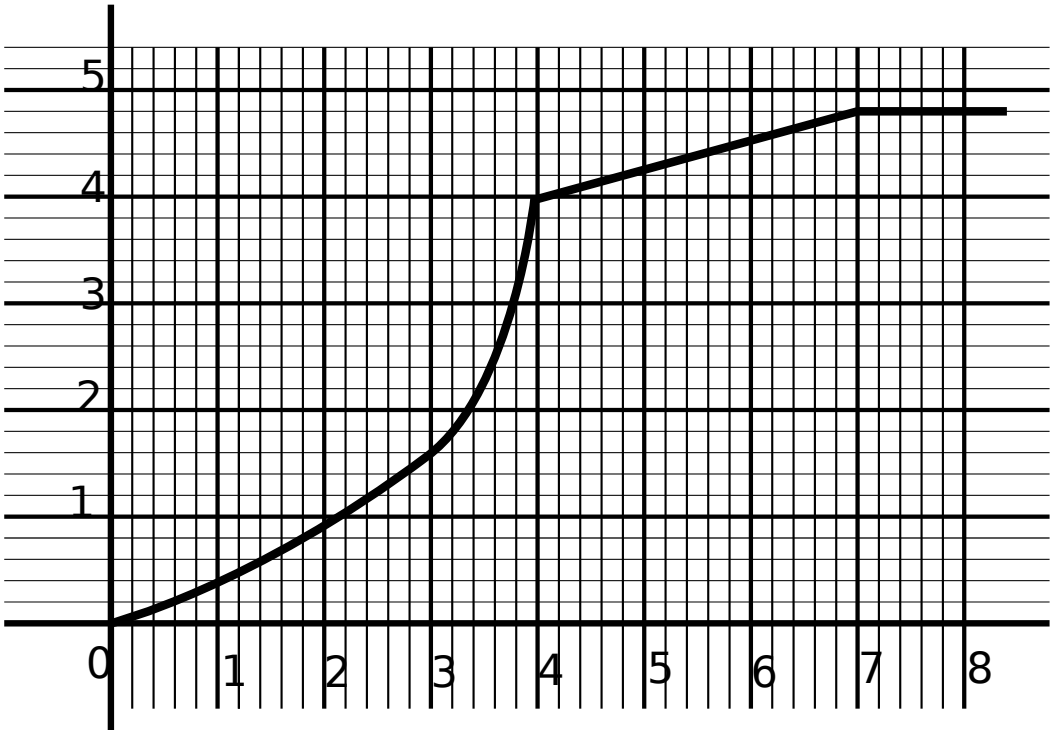
Math 220 AD9 Spring 2009 Worksheet 9

1. Golbez (a hyperactive toy fox terrier) loves playing fetch inside the house. He accelerates while he chases his toy, tries to stop when he gets to the toy, and then keeps on sliding forward on the wooden floors until he hits the wall (splat). His distance travelled, $d(t)$, at each time t is described by the following graph:



What parts of the graph correspond with each part of the story above?
When is Golbez going fastest? Slowest? What is $d(2)$, $d(4)$? How can you use this to estimate Golbez's speed at time $t = 3$? Can you find a better approximation? An even better approximation? What about his speed at time $t = 4$? Now rephrase all of this in terms of derivatives.

2. Explain (complete with beautiful pictures) how to use secant lines to approximate the slope of the tangent line to $y = x^2 + 3$ at the point $(2, 7)$. Approximate this slope by using suitable secant lines. Find the equation of the tangent line to this curve. Verify your answer by sketching the curve and its tangent line.

3. For each of the following functions, write down $f(a + h)$ for the given value of a.

   (a) $f(x) = x^2 - x$, $a = 3$
   (b) $f(x) = \frac{x^3+1}{x-1}$, $a = 0$
   (c) $f(x) = \sin \frac{x}{2}$, $a = \pi$
   (d) $f(x) = \frac{x}{x-1}$, $a = 2$.

Also write down what $\dfrac{f(x+h)-f(x)}{h}$ is for each of the above functions.

4. Find the equation of the tangent lines to the following curves at the given points:
   (a) $y = 2x^2 - 3x + 1$ at $x = 2$
   (b) $f(x) = \sqrt{x+11}$ at $x = 5$
   (c) $y = \frac{2}{2x-1}$ at $x = 2$
   (d) $f(x) = \frac{x}{x-1}$ at $x = 2$
   (e) $y = 5x + 3$ at $x = -1$ (Why should you expect this answer?)

5. Using graphs, limits, or sinister voodoo rituals, determine whether the tangent line to $y = f(x)$ exists at $x = a$. If not, why not? If so, estimate its slope.
   (a) $f(x) = \begin{cases} x^2 + x & \text{if } x < 1 \\ 3x - 1 & \text{if } x \geq 1 \end{cases}$ at $a = 1$
   (b) $f(x) = |x + 2|$ at $a = -2$.
   (c) $\begin{cases} x^2 - 1 & \text{if } x \leq 3 \\ 14 - 2x & \text{if } x > 3 \end{cases}$

6. Find the derivatives of the following functions:
   (a) $f(x) = \sqrt{4x+5}$
   (b) $f(x) = \frac{5}{3x-1}$
   (c) $f(x) = 3x^2 - x$
   (d) $f(x) = x^3$

7. Draw a curve whose tangent line at $x = -2$ intersects the curve in exactly 4 points.

8. Find the equation of the two lines through $(3, 4)$ that are tangent to the parabola $y = x^2$.

9. For each of the following functions, draw a rough sketch of the graph of that function. Using no other information besides that graph, sketch the graphs of the derivatives of the functions below.
$$e^x, \quad \ln x, \quad \sin x, \quad x^3.$$

10. Questions 39 and 40 of Section 2.1, p. 157.

11. Questions 13-24 of Section 2.2, p. 167.

Preparation for next time

Your mission should you choose to accept it (and also if you don't so choose) is to write a *good* exam question based on section
of your book.