Math 220 AD8&AD9 Practice Exam

This practice exam is intended to give you a chance to sit a midterm exam for Calc 1 in a classroom setting under time pressure without affecting your grade.

Disclaimer: The presence or absence of specific material on this practice exam is no guarantee of the presence or absence of that material on the actual midterm exam. All material covered in the course so far is fair game for the exam.

No calculators are permitted. Cell phones, MP3 players, and other recreational electronic devices must be turned off and put away.

1. (a) Given a function $f(x)$, define what it means for f to be *one to one*.

   (b) Is $f(x) = x^2$ with domain $\mathbb{R}$ invertible? Explain your answer.

   (c) Find the domain of the function $f(x) = \ln(x + 3)$.

   (d) Find the range of the function $f(x) = \ln(x + 3)$.

2. Evaluate the following limits if they exist, or explain why they do not exist. Show all of your work and use proper notation.

   (a) $\lim_{x \to 5} \dfrac{1}{x^2(x+5)}$

   (b) $\lim_{x \to -1} \dfrac{x^2 - 2x - 3}{x^2 - 5x - 6}$

   (c) $\lim_{x \to 16} \dfrac{4 - \sqrt{x}}{16 - x}$

   (d) $\lim_{x \to 1} \dfrac{1}{(x-1)^2}$

3. A velociraptor's position after t minutes of running in a straight line is given by $p(t) = 9t^2 - t^3$, $t \in [0, 3]$. Find the velociraptor's average velocity over this time interval.

4. Use the limit definition of the derivative to find $f'(x)$ for $f(x) = x^2 - 7x$. (No credit would be given for using other methods.)

5. (a) Give a graphical example of a function $f(x)$ satisfying

$$\lim_{x \to 5^+} f(x) = 3 \qquad \text{and} \qquad \lim_{x \to 5^-} f(x) = -3.$$

(b) Is the function you drew continuous at $x = 5$? Explain your answer clearly.

6. Identify and classify all discontinuities of the following function. Justify your answers using limits.

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{for } x \geq 1, \\ \frac{1}{x} & \text{for } x < 1. \end{cases}$$

7. Use the Intermediate Value Theorem to prove that the function $f(x) = x^7 - 7x^2 + 5$ has a zero in the interval $[0, 1]$.