Math 241, Spring 2007, Merit Practice Exam 2

1. Let $f(x,y,z) = \sqrt{xy^2z^3}$ and let P be the point $(2,2,2)$.

   (a) Find the maximum directional derivative of f at P and the direction in which it occurs.

   (b) Find the directional derivative of f at P in the direction of $\vec{v} = 3\vec{i} + 12\vec{j} + 4\vec{k}$.

2. Find and classify the critical points of the function $f(x,y) = 4xy - 2x^4 - y^2$.

3. Find the first octant point on the surface $xyz = 8$ that is closest to $(0,0,0)$. (First octant = x,y,z all positive).

4. Find the equation of the tangent plane to the surface $xy^2 + 2xyz - e^{xz} = 8$ at the point $(1,3,0)$.

5. Use linear approximation to estimate

$$\sqrt{(3.1)^2 + (4.2)^2 + (11.7)^2}$$

6. Find the highest point on the surface $z = 4xy - x^4 - y^4$.

7. Suppose that $r = uvw - u^2 - v^2 - w^2$, $u = y + z$, $v = x + z$, $w = x + y$. Find $\frac{\partial r}{\partial x}$.

8. Find $\frac{\partial z}{\partial x}$ supposing that $z = f(x,y)$ satisfies the equation $xyz = \sin(x + y + z)$

9. Show that the sphere $x^2 + y^2 + z^2 = r^2$ and the cone $z^2 = a^2x^2 + b^2y^2$ are orthogonal (that is, have perpendicular tangent planes) at every point of their intersection. (Fig. 12.8.12).

10. Find the maximum and minimum values that the function $f(x,y,z) = 3x + 2y + z$ attains on the surface $x^2 + y^2 + z^2 = 1$.