Merit Math 241, Practice Exam 3, Spring 2007

Usual warnings apply: I am not writing the exam and I have not seen the exam.

1. Evaluate
$$\int_0^8 \int_{x^{2/3}}^4 x \cos y^4 \, dy \, dx.$$

2. Find the area of that section of the surface $z = x + y^2$ that lies above $0 \leq x \leq y$, $0 \leq y \leq 2$.

3. Substitute $u = xy$ and $v = xy^3$ to find the area of the first quadrant region bounded by the curves $xy = 2$, $xy = 4$, $xy^3 = 3$, $xy^3 = 6$.

4. Find the volume of the solid bounded by the paraboloids $z = 2x^2 + 2y^2$ and $z = 48 - x^2 - y^2$.

5. Convert to rectangular coordinates: $\int_{\pi/2}^{3\pi/2} \int_{1/2}^{2} \frac{1}{r^3} \, dr \, d\theta$.

6. Convert to cylindrical coordinates: $\int_1^5 \int_{-\sqrt{25-x^2}}^{\sqrt{25-x^2}} xy^2 \, dy \, dx$.

7. Set up the integrals but do not evaluate:

   Consider a lamina that occupies the region D bounded by the parabola $x = 1 - y^2$ and the coordinate axes in the first quadrant with density function $\delta(x, y) = y$.

   (a) Find the mass of the lamina.

   (b) Find the centroid.

   (c) Find the polar moment of inertia.

8. Set up but do not evaluate the triple integrals:

   (a) the triple integral for the volume of the solid bounded by the planes $z = 0$, $z = 20 - x$ and the cylinder $x^2 + y^2 = 25$.

   (b) the mass and centroid of a tetrahedron with density $\delta(x, y, z) = xy + z^2$, where the tetrahedron lies in the first octant, bounded by the coordinate axes and the plane $x + y + z = 1$.

   (c) the triple integral for the volume of the solid bounded by $z = x^2$, $y + z = 4$, $y = 0$, $z = 0$.