Math 241, Spring 2007, Merit Worksheet 12

1. Find an equation for the plane tangent to the paraboloid $z = 2x^2 + 3y^2$ and, simultaneously, parallel to the plane $4x - 3y - z = 10$.

2. Find the absolute maximum and minimum values attained by the function $f(x, y) = 3x^2y - 3xy - x + 2$ on the triangle with vertices $(0, 0)$, $(0, 2)$ and $(4, 0)$.

3. The function $f(x, y) = x^3 + 12xy + y^4$ has

   (a) No global maximum or minimum

   (b) A global max but no global min

   (c) A global min but no global max

   (d) Both a global max and a global min

4. Suppose that the function f is continuous on the disk D bounded by the unit circle $x^2 + y^2 = 1$. Is it possible that $f(x, y)$ attains both its maximum and its minimum values on D at points of the boundary circle? Illustrate your answer with an example.

5. Prof Nevin's cardboard box factory has an order for open-topped boxes with a volume of 600 in^3. The material for the bottom of the box costs 6¢$/in^2$ and the material for its sides costs 5¢$/in^2$. What are the dimensions of the box that is most economical to manufacture?

6. Find the first octant point $P(x, y, z)$ on the plane $2x + 3y + z = 49$ which is closest to the point $Q(7, -7, 0)$.

7. A very long rectangle of sheet metal has width L and is to be folded to make a rain gutter. Maximize its volume by maximizing the cross-sectional area, as shown.

8. Consider $z = 2x^2 + 8xy + y^4$. Does this surface open upwards or downwards? What is its highest/lowest point?

9. Find and classify all the critical points of the function $f(x, y) = x^3 + 6xy + 3y^2 - 9x$.

10. Consider the function $f(x, y) = \frac{1}{2}(ax^2 + by^2)$.
    (a) Show that $(0, 0)$ is a critical point.
    (b) For what values of a and b does f have a maximum at $(0, 0)$? What does the surface look like?
    (c) For what values of a and b does f have a minimum at $(0, 0)$? What does the surface look like?
    (d) For what values of a and b does f have a saddle point at $(0, 0)$? What does the surface look like?

11. Which of the following guarantees a saddle point of the function $f(x, y)$ at the point (a, b)?
    (a) f_{xx} and f_{yy} have the same sign at (a, b)
    (b) f_{xx} and f_{yy} have different signs at (a, b)
    (c) f_{xy} is negative at (a, b)
    (d) None of the above.

12. Which of the following would be enough evidence to conclude that $f(x, y)$ has a global minimum?
    (a) D is always positive
    (b) $f_{xx} > 0$ and $f_{yy} > 0$
    (c) $f(x, y)$ has no saddle point and no local maxima
    (d) None of the above.

13. Find and classify the critical points of the functions $f(x, y) = x^4 + y^4$, $g(x, y) = x^3 + y^3$.

Warm-Up Problems for Next Time

1. Use the method of Lagrange multipliers to find the maximum and minimum values (if any) of the function $f(x, y) = x + y$ subject to the constraint $x^2 + 4y^2 = 1$.