Math 241, Spring 2007, Merit Worksheet 13

Lagrange Multipliers

1. Find the maximum and minimum values of $f(x, y) = x^2 + 2y^2$ subject to the constraint $x^2 = 1 - y^2$.
2. Find the point or points of the surface $z = xy + 5$ closest to the origin.
3. Find the highest and lowest point on the ellipse formed by the intersection of the cone $z^2 = x^2 + y^2$ and the plane $x + 2y + 3z = 3$.

Critical Points

4. Find and classify all the critical points of the function $f(x, y) = x^3 + 6xy + 3y^2 - 9x$.
5. Consider the function $f(x, y) = \frac{1}{2}(ax^2 + by^2)$.
   (a) Show that $(0, 0)$ is a critical point.
   (b) For what values of a and b does f have a maximum at $(0, 0)$? What does the surface look like?
   (c) For what values of a and b does f have a minimum at $(0, 0)$? What does the surface look like?
   (d) For what values of a and b does f have a saddle point at $(0, 0)$? What does the surface look like?
6. Which of the following guarantees a saddle point of the function $f(x, y)$ at the point (a, b)?
   (a) f_{xx} and f_{yy} have the same sign at (a, b)
   (b) f_{xx} and f_{yy} have different signs at (a, b)
   (c) f_{xy} is negative at (a, b)
   (d) None of the above.
7. Which of the following would be enough evidence to conclude that $f(x, y)$ has a global minimum?

(a) Δ is always positive

(b) $f_{xx} > 0$ and $f_{yy} > 0$

(c) $f(x, y)$ has no saddle point and no local maxima

(d) None of the above.

8. Find and classify the critical points of the functions $f(x, y) = x^4 + y^4$, $g(x, y) = x^3 + y^3$.

9. A wire 120 cm long is cut into three pieces of length x, y, and $120-x-y$ and each piece is bent into the shape of a square. Let $f(x, y)$ denote the sum of the areas of the these squares. Show that the single critical point of f is a local minimum. But surely it is possible to maximize the sum of these areas. Explain.

10. Find and classify the critical points of the function

$$f(x, y) = \sin\frac{\pi x}{2}\sin\frac{\pi y}{2}$$

Some review

11. Suppose $f(x, y) = \frac{xy}{x^2+y^2}$.

    (a) Find ∇f.

    (b) Find the equation of the tangent plane to f at the point $(3, 4, 12/5)$.

    (c) Find df.

    (d) Estimate the value of f at $(2.8, 4.1)$.

    (e) What is the directional derivative of f at the point $(4, 3)$ in the direction of $5\vec{i} + 12\vec{j}$?

Warm-up for next time

1. Practice Exam Wednesday at 7pm in the Merit room. Office hours Wednesday at 4pm or email me to make an appointment. I'm here to help.