

## Math 241, Spring 2007, Merit Worksheet 14

### Some review

1. Suppose  $f(x, y) = \frac{xy}{x^2+y^2}$ .
  - (a) Find  $\nabla f$ .
  - (b) Find the equation of the tangent plane to  $f$  at the point  $(3, 4, 12/25)$ .
  - (c) Find  $df$ .
  - (d) Estimate the value of  $f$  at  $(2.8, 4.1)$ .
  - (e) What is the directional derivative of  $f$  at the point  $(4, 3)$  in the direction of  $5\vec{i} + 12\vec{j}$ ?
2. An open-topped rectangular box is to have total surface area  $300 \text{ cm}^2$ . Find the dimensions that maximize its volume.
3. The aggregate resistance  $R$  of three variable resistances  $R_1, R_2$ , and  $R_3$  connected in parallel satisfies the harmonic equation  $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ . Suppose that  $R_1$  and  $R_2$  are  $100\Omega$  and are increasing at  $1\Omega/s$  while  $R_3$  is  $200\Omega$  and is decreasing at  $2\Omega/s$ . Is  $R$  increasing or decreasing at that instant? At what rate?

### Chapter 13: Double Integrals

4. Without doing any calculations:
  - (a) Let  $R$  be the square defined by  $-1 \leq x \leq 1, -1 \leq y \leq 1$ . Is the sign of  $\int \int_R x^4 dA$  positive, negative or zero?
  - (b) Is  $\int \int_R y^3 dA$  positive, negative or zero?
5. If  $[[x]]$  denotes the greatest integer less than  $x$  (e.g.,  $[[2.7]] = 2$ ), evaluate the integral  $\int \int_R [[x + y]] dA$ , where  $R$  is the region  $\{(x, y) \mid 1 \leq x \leq 3, 2 \leq y \leq 5\}$ .

### Warm-Up for next time

1. Find  $\int \int_D 3 dA$ , where  $D$  is the circular region in the  $xy$ -plane with centre  $C(10, -50)$  and radius 2. What if the centre is at  $(-1, 15)$ ?