

Math 241, Spring 2007, Merit Worksheet 16

1. Evaluate $\int_0^1 \int_0^1 e^{\max(x^2, y^2)} dx dy$, where $\max(x^2, y^2)$ means the larger of the two numbers x^2 and y^2 .
2. Switch the order of integration:

$$\int_{-1}^1 \int_{(x+1)^2}^{2x+2} f(x, y) dy dx.$$

3. Use polar coordinates to combine the sum

$$\int_{1/\sqrt{2}}^1 \int_{\sqrt{1-x^2}}^x xy dy dx + \int_1^{\sqrt{2}} \int_0^x xy dy dx + \int_{\sqrt{2}}^2 \int_0^{\sqrt{4-x^2}} xy dy dx$$

into one double integral and then evaluate.

4. Convert to polar coordinates and evaluate:

$$\int_{-2}^0 \int_0^{\sqrt{4-x^2}} x^2 + y^2 dy dx.$$

5. Express the following integral in cartesian coordinates:

$$\int_0^3 \int_{\pi}^{\pi/2} r d\theta dr.$$

6. Evaluate the following integral:

$$\int \int_R \sqrt{x^2 + y^2} dA,$$

where R is the region $R = \{(x, y) \mid 1 \leq x^2 + y^2 \leq 9, y \geq 0\}$.

7. The Archimedean spiral is the equation $r = \theta$. What area is enclosed by the spiral when θ ranges from 0 to 3π ?
8. Find the volume of the region bounded by the paraboloids $z = x^2 + 2y^2$ and $z = 12 - 2x^2 - y^2$.
9. Find the volume of the “ice-cream cone” bounded by the sphere $x^2 + y^2 + z^2 = 4$ and the cone $z = \sqrt{x^2 + y^2}$

10. Evaluate

$$\int_0^\infty \int_0^\infty \frac{1}{(1+x^2+y^2)^2} dx dy.$$

11. Suppose that D is the disk with $x^2+y^2 \leq 1$. Find (without integrating)

(a) $\iint_D \sqrt{1-x^2-y^2} dA$

(b) $\iint_D 5 - x^2 \sin x + y^3 \cos y dA$

(c) $\iint_D (1 - \sqrt{x^2+y^2}) dA.$

12. Suppose that R is the region in the xy -plane between the two curves $y = 2x^2$ and $y = 2x - 4$. Find the volume of the region between the surface $z = x$ and the region R .

Warm-Up for next time

1. Find the volume of the solid that lies below the surface $z = x^2$ and above the region in the xy -plane bounded by the curves $y = x^2$ and $y = 1$.