Math 241, Spring 2007, Merit Worksheet 17

1. Suppose that D is the disk with $x^2+y^2 \leq 1$. Find (without integrating)
   (a) $\int\int_D \sqrt{1-x^2-y^2}\ dA$
   (b) $\int\int_D 5 - x^2 \sin x + y^3 \cos y\ dA$
   (c) $\int\int_D (1-\sqrt{x^2+y^2})\ dA$.

2. Evaluate
$$\int_0^\infty \int_0^\infty \frac{1}{(1+x^2+y^2)^2}\ dx\ dy.$$

3. Use double integrals to find the volume of the solid that lies inside the ellipsoid $4x^2+4y^2+z^2=8$ and above the paraboloid $z=2x^2+2y^2$.

4. Use a double integral to evaluate the area enclosed by one leaf of the four-leaved rose $r=\cos 2\theta$.

5. Find the centroid of the rectangular lamina $\{-1 \leq x \leq 1, -1 \leq y \leq 1\}$ with density $\delta(x,y) = 2y + x^2 + 1$. Can you exploit symmetry in any way?

6. Find the mass and centroid of the region bounded by $y=0, x=-1, x=1$, and $y=e^{-x^2}$, with density $\delta(x,y)=|xy|$.

7. Consider a disc D with density $\delta(x,y) = \frac{1}{\sqrt{x^2+y^2}}$, centred at the origin and of radius a.
   (a) Find the moments of inertia I_x, I_y, I_0.
   (b) If it is rotating at $4 rad/sec.$, what is KE_{ROT}?
   (c) Where should I place a very small but very dense weight of the same mass as the disc, so that when rotated about the z-axis, it will have the same kinetic energy as the rotating disc?

8. Find the mass and centroid of the region inside the circle $r=2\sin\theta$ and outside the circle $r=2$ with density $\delta(x,y)=y$.

9. Find the polar moment of inertia of the right-hand loop of the lemniscate $r^2=\cos 2\theta$, with density $\delta(x,y)=r^2$. (See Figure 13.5.21).

10. A uniform rectangular plate with base length a, height b, and mass m is centred the origin. Show that its polar moment of inertia is $I_0 = \frac{1}{12}m(a^2 + b^2)$.

11. What region T in $\mathbb{R}^3$ maximizes the volume of the integral

$$\int\int\int_T (1 - x^2 - y^2 - z^2)\ dV?$$

12. Describe the region of integration for the following integral:

$$\int_{-1}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}}$$

Warm-Up for next time

1. Compute the value of the triple integral

$$\int\int\int_T x^2\ dV,$$

where T is the tetrahedron bounded by the coordinate planes and the first octant part of the plane with equation $x + y + z = 1$.