

Math 241, Spring 2007, Merit Worksheet 18

1. Find the polar moment of inertia of the disk bounded by $r = 2 \cos \theta$ with density $\delta(x, y) = k$, a positive constant.
2. Describe the region of integration for the following integral:

$$\int_{-1}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} \sqrt{x^2 + y^2 + z^2} dz dy dx$$

3. Evaluate the integral over the region $E = \{(x, y, z) \mid 0 < x < 1, 0 < y < x, x \leq z \leq 2x\}$.

$$\int \int \int_E yz \cos x^5 dV$$

4. Consider the integral $\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} dz dy dx$.
 - (a) What does this integral evaluate for us?
 - (b) Sketch the region of integration.
 - (c) Rewrite the integral in the following orders:

$$dx dy dz, \quad dz dx dy$$

- (d) Evaluate the integral.
5. Set up the triple integral to find the volume of the solid bounded by the parabolic cylinder $x = y^2$ and the planes $z = 0, x + z = 1, x = 1$.
 6. Set up the integrals necessary to find the centroid of the solid bounded by the surfaces $z = 1 - y^2, x + z = 2, x = 0, z = 0$.
 7. Evaluate

$$\int \int \int_E xz dV,$$

where E is bounded by the planes $z = 0, z = y$ and the cylinder $x^2 + y^2 = 1$.

8. Let E be the solid region bounded by the lower half-cone $z = -2\sqrt{x^2 + y^2}$ and the two spheres $x^2 + y^2 + z^2 = 1, x^2 + y^2 + z^2 = 4$. Express the volume of E as a triple integral in spherical coordinates.

9. Let F be the solid region bounded below by the xy -plane, bounded above by the paraboloid $z = 9 - x^2 - y^2$, and on the sides by the cylinder $x^2 + y^2 - 2x = 0$. Express the volume of F as a triple integral in cylindrical coordinates.
10. What domain D in space maximizes the value of the integral

$$\int \int \int_D (1 - x^2 - y^2 - z^2) dV?$$

Warm-Up for next time

1. Convert to spherical coordinates:

$$\int \int \int_E (x^2 + y^2 + z^2)^{3/2} dV$$

where E is the solid bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 9$.