Math 241, Spring 2007, Merit Worksheet 19

1. Rewrite the following integral using cylindrical coordinates:

$$\int_0^2 \int_{-\sqrt{4-x^2}}^0 \int_{-\sqrt{9-x^2-y^2}} xz + x^2 + y^2 \; dz \; dy \; dx.$$

2. Find the volume of the region bounded by the surfaces $z = y$, $z = x^2$, $y = 4$ and $z = 0$.

3. Set up the triple integral to find the volume of the solid bounded by the parabolic cylinder $x = y^2$ and the planes $z = 0, x + z = 1, x = 1$.

4. Set up the integrals necessary to find the centroid of the solid bounded by the surfaces $z = 1 - y^2, x + z = 2, x = 0, z = 0$ with density $\delta(x, y, z) = 2 - z$.

5. Evaluate

$$\int\int\int_E xz \; dV,$$

where E is bounded by the planes $z = 0$, $z = y$ and the cylinder $x^2 + y^2 = 1$.

6. Set up integrals to find the volume of the wedge above the xy-plane cut from the cylinder $x^2 + 9y^2 = 4$ by the planes $z = 0$ and $z = 3x$. Rewrite the integral in a different order of integration.

7. Let F be the solid region bounded below by the xy-plane, bounded above by the paraboloid $z = 9 - x^2 - y^2$, and on the sides by the cylinder $x^2 + y^2 - 2x = 0$. Express the volume of F as a triple integral in cylindrical coordinates.

8. Find the volume of the region that lies inside both the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 - 2x = 0$.

9. What domain D in space maximizes the value of the integral

$$\int\int\int_D (1 - x^2 - y^2 - z^2) \; dV?$$

10. Let E be the solid region bounded by the lower half-cone $z = -2\sqrt{x^2 + y^2}$ and the two spheres $x^2 + y^2 + z^2 = 1$, $x^2 + y^2 + z^2 = 4$. Express the volume of E as a triple integral in spherical coordinates.

11. Use a change of variables to evaluate

$$\int\int\int_E x^2 y \; dV,$$

where E is the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1.$$

12. Find a transformation that maps the unit square to a parallelogram with vertices $(0, 0), (1, 2), (2, 1)$ and $(3, 3)$.

13. Find the area of the region R inside the ellipse $(2x+y)^2+(x+3y)^2 = 16$. (Hint: Use a suitable change of variables.)

Warm-Up for next time

1. Convert to spherical coordinates:

$$\int\int\int_E (x^2 + y^2 + z^2)^{3/2} \; dV$$

where E is the solid bounded by the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 9$.