Math 241, Spring 2007, Merit Worksheet 2

1. Let $\vec{a} = -4\mathbf{i} + 6\mathbf{j}$, $\vec{b} = 1\mathbf{i} + 3\mathbf{j}$, $\vec{c} = 3\mathbf{i} + 2\mathbf{j}$. Sketch these vectors on a single graph.

   (a) Calculate $\vec{a} \cdot \vec{b}$, $\vec{a} \cdot \vec{c}$.

   (b) Do you notice anything?

   (c) Find $\vec{b} \cdot \vec{b}$, $\vec{a} \cdot \vec{a}$.

   (d) Find $|\vec{a}|$, $|\vec{b}|$.

2. Let $\vec{a} = 3\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ and $\vec{b} = \mathbf{i} + 6\mathbf{j} + \mathbf{k}$ and $\vec{c} = 4\mathbf{i} - 7\mathbf{j} + 2\mathbf{k}$. (We could also write $\vec{a} =< 3, 7, 2 >$.)

   (a) Calculate $\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$.

   (b) Calculate $(\vec{a} + \vec{b}) \cdot \vec{c}$.

   (c) Calculate $(2\vec{a}) \cdot \vec{b}$, $2(\vec{a} \cdot \vec{b})$.

   (d) Prove these patterns always hold.

3. For what values of t are $< t, 1, t-1 >$ and $< 2, -4, 1 >$ perpendicular?

4. Find the angle θ between the following pair of vectors:

   $12\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ and $8\mathbf{i} - 15\mathbf{j}$.

   Find the angle between $\vec{x} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and

   (a) the x-axis.

   (b) the y-axis.

   (c) Find the unit vector $\frac{\vec{x}}{|\vec{x}|}$.

   (d) Quickly say what the angle between $\vec{x}$ and the z-axis is. Explain.

   (e) What are the direction cosines of $\vec{x}$?

5. Suppose that $\vec{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\vec{b} = \mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$.

   (a) Find $comp_{\vec{a}}\ \vec{b}$, the component of $\vec{b}$ along $\vec{a}$.

   (b) Find $proj_{\vec{a}}\ \vec{b}$, the projection of $\vec{b}$ onto $\vec{a}$.

   (c) Draw a sketch showing all of these.

6. Suppose that the horizontal and vertical components of the three vectors shown balance. How much work is done by the constant force $\mathbf{F}$ (parallel to the inclined plane) in pulling the weight mg up the inclined plane a vertical height h?

7. Calculate $(-\mathbf{i} + 4\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$.

8. Find the area of the triangle in $\mathbb{R}^3$ between the points $A = (1, 1, 1)$, $B = (4, 5, 2)$ and $C = (6, 3, 8)$.

9. The cross product:

   (a) Consider the vector $\vec{c} = \vec{a} \times (\vec{a} \times \vec{b})$. Is $\vec{c} \perp \vec{a}$? Is $\vec{c} \perp \vec{b}$? Prove or give a counterexample.

   (b) Suppose that $\mathbf{j} \times \vec{a} = \mathbf{i}$. Give two possible solutions for $\vec{a}$ and discuss the other possible solutions.

10. A parallelepiped is a 3-d object with parallelograms for side, like a tilted box. Its volume is the area of its base times its height (in the direction perpendicular to that base). The three vectors $\vec{a} = 6\mathbf{i}+2\mathbf{j}+\mathbf{k}$, $\vec{b} = \mathbf{i}+5\mathbf{j}$, $\vec{c} = 2\mathbf{i}+2\mathbf{j}+7\mathbf{i}$ determine a parallelepiped. Find its volume. Find a formula for the volume of a parallelepiped with sides $\vec{x}, \vec{y}, \vec{z}$.

Warm-Up Problems for Next Time

1. Find the equation of the plane through the point $(1, -3, 2)$ with normal vector $2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$.