

Math 241, Spring 2007, Merit Worksheet 20

Cylindrical Coordinates

1. Find the volume of the region bounded below by the paraboloid $z = x^2 + y^2$ and above by the plane $z = 2x$.

Change of Variables

2. Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ if
 - (a) $u = x + y, v = 2x - 3y$
 - (b) $u = x^2 - y^2, v = y/x$.
3. Find the area of the first quadrant region bounded by the curves $y = x^2, y = 2x^2, x = y^2$ and $x = 4y^2$.
4. Find the area of the region R inside the ellipse $(2x+y)^2 + (x+3y)^2 = 16$. (Hint: Use a suitable change of variables.)

Spherical Coordinates

5. Let E be the solid region bounded by the lower half-cone $z = -2\sqrt{x^2 + y^2}$ and the two spheres $x^2 + y^2 + z^2 = 1, x^2 + y^2 + z^2 = 4$. Express the volume of E as a triple integral in spherical coordinates.
6. Describe the surface $\rho = 1 + \cos \phi$ and compute the volume of the region it bounds.
7. Find the volume and centroid of the uniform solid that lies inside the sphere $\rho = a$ and above the cone $r = z$.

Some more of the above

8. Let F be the solid region bounded below by the xy -plane, bounded above by the paraboloid $z = 9 - x^2 - y^2$, and on the sides by the cylinder $x^2 + y^2 - 2x = 0$. Express the volume of F as a triple integral in cylindrical coordinates.

9. Find the volume of the region that lies inside both the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 - 2x = 0$.
10. Use elliptical coordinates $x = 3r \cos \theta$, $y = 2r \sin \theta$ to find the volume of the region bounded by the xy -plane, the paraboloid $z = x^2 + y^2$ and the elliptic cylinder
$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$
11. Find a transformation that maps the unit square to a parallelogram with vertices $(0, 0)$, $(1, 2)$, $(2, 1)$ and $(3, 3)$.
12. A spherical shell of mass m is bounded by the spheres $\rho = a$ and $\rho = 2a$. Its density is given by $\delta = \rho^2$. Find its moment of inertia about a diameter.
13. Find the average distance of points of a solid ball of radius a from the center of the ball.

Warm-Up for next time

1. Hour Exam 3 on Wednesday at 9am.
2. Practice Exam Monday at 7pm in the Merit Room.
3. Office Hour: Monday at 3pm in the Merit Room.
4. Extra Office Hours.