Math 241, Spring 2007, Merit Worksheet 21

1. Find the area of the part of the surface $2z = x^2$ that lies directly above the triangle in the xy-plane with vertices at $(0, 0)$, $(1, 0)$ and $(1, 1)$.

2. Find the area that is cut from the surface $z = x^2 - y^2$ by the cylinder $x^2 + y^2 = 4$.

3. Show by integration that the surface of the conical surface $z = br$ between the planes $z = 0$ and $z = h = ab$ is given by $A = \pi aL$ where L is the slant height $\sqrt{a^2 + h^2}$ and a is the radius of the base of the cone.

4. Let F be the solid region bounded below by the xy-plane, bounded above by the paraboloid $z = 9 - x^2 - y^2$, and on the sides by the cylinder $x^2 + y^2 - 2x = 0$. Express the volume of F as a triple integral in cylindrical coordinates.

5. Convert to spherical and evaluate:

$$\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} \left(x^2 + y^2 + z^2\right)\, dz\, dx\, dy.$$

6. Find the volume of the solid region that remains in the spherical solid $\rho \leq 4$ after the solid cone $\phi \leq \pi/6$ has been removed.

7. Use elliptical coordinates $x = 3r\cos\theta$, $y = 2r\sin\theta$ to find the volume of the region bounded by the xy-plane, the paraboloid $z = x^2 + y^2$ and the elliptic cylinder
$$\frac{x^2}{9} + \frac{y^2}{4} = 1.$$

8. Find a transformation that maps the unit square to a parallelogram with vertices $(0, 0), (1, 2), (2, 1)$ and $(3, 3)$.

9. A spherical shell of mass m is bounded by the spheres $\rho = a$ and $\rho = 2a$. Its density is given by $\delta = \rho^2$. Find its moment of inertia about a diameter.

10. Find the average distance of the points of a solid ball of radius a from the center of the ball.

Warm-Up for next time

1. Hour Exam 3 tomorrow at 9am. Go to Altgeld 447.
2. I will be in my office (150 Altgeld) until 5pm if you have any questions.