Math 241, Spring 2007, Merit Worksheet 23

1. Evaluate
$$\int_C x^2y^2 + z^2\ ds$$
where C is the curve $\mathbf{r}(t) = 3\vec{i} + \sin 2t\vec{j} + \cos 2t\vec{k}$, for $0 \leq t \leq \pi$. Now calculate this integral if C is the curve $\mathbf{r}(t) = 3\vec{i} + \sin 2(\pi - t)\vec{j} + \cos 2(\pi - t)\vec{k}$ for $0 \leq t \leq \pi$.

   What is the connection between these two curves?

2. Evaluate
$$\int_C y\sqrt{x}\ dx + x\sqrt{x}\ dy,$$
where C is the part of the graph $y^2 = x^3$ from $(-1, -1)$ to $(1, 1)$. Now calculate $\int_{-C} y\sqrt{x}\ dx + x\sqrt{x}\ dy$.

3. Evaluate
$$\int_C \mathbf{F} \cdot \mathbf{T}\ ds$$
where $\mathbf{F}(x, y, z) = xyz\vec{i} + x^3y^5z^6\vec{j} + z\vec{k}$ and C is the curve $\mathbf{r}(t) = t^2\vec{i} + 7\vec{j} + t\vec{k}$ for $0 \leq t \leq 3$.

4. Calculate
$$\int_C \mathbf{F} \cdot \mathbf{T}\ ds$$
where $\mathbf{F}(x, y, z) = (2x + 3y)\vec{i} + (3x + 2y)\vec{j} + 3z^2\vec{k}$ and C is the path from $(0, 0, 0)$ to $(4, 2, 3)$ that consists of three line segments parallel to the x-axis, the y-axis and the z-axis, in that order.

5. Check if the vector field is conservative and find its potential:

   (a) $\mathbf{F}(x, y) = e^{x-y}\left((xy + y)\vec{i} + (xy + x)\vec{j}\right)$

   (b) $\mathbf{G}(x, y) = (\cos x + \log y)\vec{i} + (\frac{x}{y} + e^y)\vec{j}$

6. Show that the line integral below is independent of path and then calculate the value of the line integral:
$$\int_{(0,0)}^{(1,-1)} (e^y + ye^x)\ dx + (e^x + xe^y)\ dy.$$

7. Show that if **F** is a constant force field, then it does zero work on a particle that moves once uniformly counterclockwise around the unit circle in the xy-plane.

8. Suppose that $\mathbf{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\mathbf{r}|$. Show that $\nabla \cdot \frac{\mathbf{r}}{r} = 0$.

9. Suppose that F is a differentiable vector field and that f is a differentiable scalar function. Show that

$$\nabla \cdot (f\mathbf{G}) = f\nabla \cdot \mathbf{G} + (\nabla f) \cdot \mathbf{G}$$

10. Show that *div* (*curl* **F**) $= 0$, if the vector field **F** is twice continuously differentiable.

11. Problems 30 and 31 p. 1039

Warm-up for next time

Problem 21 p.1039