Math 241, Spring 2007, Merit Worksheet 25

1. Use Green's Theorem to evaluate the line integral

$$\int_C (1 + y + e^{-x^2})\ dx\ \ +\ \ (2x + \cos y^5)\ dy,$$

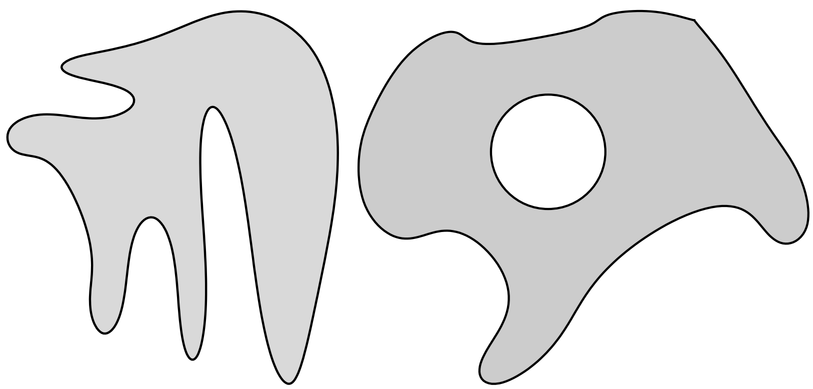
where C is the curve obtained by going around the square $\{(x, y) : -1 \leq x \leq 1, -1 \leq y \leq 1$ in a counterclockwise direction.

2. Use (a corollary of) Green's Theorem to find the area of the region

   (a) the circle bounded by $x = a\cos t, y = a\sin t,\ 0 \leq t \leq 2\pi$.

   (b) the region between the graphs of $y = x^3$ and $y = x^4$.

3. Use the pictures below to indicate the direction of the positively oriented boundary curve of the regions. Also indicate the direction of the outer normal vector.



4. Evaluate (using Green's Theorem)

$$\int_C xy\ dx + x^2\ dy,$$

where C is the curve going *clockwise* around the first-quadrant loop of the graph of the polar equation $r = \sin 2\theta$.

5. Calculate (using Green's Theorem) the outward flux of the vector field $\mathbf{F} = (3x + \sqrt{1 + y^2})\vec{i} + (2y - \sqrt[3]{1 + x^4})\vec{j}$ across the positively oriented curve C, which is the triangle with vertices $(0, 0), (3, 0)$ and $(0, 6)$.

6. Calculate (using Green's Theorem) the total circulation of the vector field $\mathbf{F} = -y^3\vec{i} + x^3\vec{j}$ around the positively oriented curve C, which is the circle with equation $x^2 + y^2 = 25$. (If he has not yet used the term "circulation" in class, the answer here is the same as if we find the work done by $\mathbf{F}$ in moving the particle once around the curve.)

7. Given a small cube resting on the xy plane with corners at $(0,0,0)$, $(a,0,0)$, $(a,a,0)$, and $(0,a,0)$, which vector field will produce positive flux through that cube?

   (a) $3\vec{i}$

   (b) $x\vec{i} - y\vec{j}$

   (c) $2\vec{i}3\vec{j} + \vec{k}$

   (d) $z\vec{k}$

8. Suppose that $f(x,y)$ is a continuously differentiable (scalar) function. Then $curl\nabla f$ is: Negative? Positive? Zero? Depends on f?

9. Find the area of the region bounded by the astroid with parametric equations $x = \cos^3 t, y = \sin^3 t$, $0 \le t \le 2\pi$.

10. Evaluate

$$\int_C (x - y)\ dx + y\ dy$$

where C is the negatively oriented boundary of the region between the $x - axis$ and the graph of $y = \sin x$ for $0 \le x \le \pi$.

Warm-up for next time

Evaluate the surface integral $\int\int_S (x + y)\ dS$, where S is the the first octant part of the plane $x + y + z = 1$.

This week's office hours will be held in my office (150 Altgeld) rather than in the Merit Room (173 Altgeld)