Math 241, Spring 2007, Merit Worksheet 3

1. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$ be vectors. Then

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

   Can you give a reason why $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| = |(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}|$ holds (without doing any calculations)?

2. (a) Find the parametric and symmetric (implicit) equations of the line through the points $P(-1, 0, 2)$ and $Q(2, 1, 1)$.

   (b) Find the midpoint of PQ and check that it satisfies the equation of the line.

3. Find the equation of the plane through the points $P(7, 2, 1)$, $Q(6, -1, 3)$ and $R(9, 3, 2)$.

4. Find the angle θ between the planes with equations

$$3x - 12y + 4z = 12 \qquad \text{and} \qquad 7x - 4y - z = 11$$

   Write symmetric equations for their line of intersection L.

5. Find the line through $(-1, 2, 3)$ which is perpendicular to the plane $3x - y + 4z = 9$.

6. You are given a line and a plane

$$3x - y + 4z = 9$$
$$x = -2t + a, \qquad y = 6t + 7a, \qquad z = 3t + 4a.$$

   Show the line is parallel to the plane. For what value of a does the line lie in the plane?

7. Let the plane H be given by the equation $x + y + z = 10$. Write down the equation of a sphere of radius 2 which is tangent to the plane at some point.

8. Show that the points $A(1, 0, 3), B(3, -2, 9), C(-2, -3, 0), D(4, 1, -1)$ are coplanar.

9. Find the equation of the plane through $P(1, -1, 1)$ that intersects the xy-plane in the same line as does the plane $3x + 2y - z = 6$.

10. Two nonparallel non-intersecting lines are said to be **skew lines**. Show that the lines L_1 and L_2 given below are skew lines:

$$\begin{aligned} L_1: &\quad x = 4 + 2t, \quad y = -5 + 4t, \quad z = 1 - 3t \\ L_2: &\quad x = 2 + t, \quad y = -1 + 3t, \quad z = 2t \end{aligned}$$

11. A line L has symmetric equations

$$\frac{2x - 5}{1} = \frac{y - 3}{-2} = \frac{z + 4}{3}$$

    (a) By considering just the first inequality what do we obtain?

    (b) Similarly find two more. Is there anything special about them?

    (c) What is the connection between these and line L?

12. What cases can arise when you have two lines L_1 and L_2 in the xy-plane? What does this tell us about two linear equations $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ in two unknowns x and y?

13. What are the possible configurations for

    (a) Three lines in $\mathbb{R}^3$?

    (b) Three planes in $\mathbb{R}^3$?

    What does this tell you about solutions of suitable systems of linear equations?

Warm-Up Problems for Next Time

1. Suppose $\mathbf{r}(t) = 3\mathbf{i}\cos 2\pi t + 3\mathbf{j}\sin 2\pi t$. Find $\mathbf{r}'(7/4), \mathbf{r}''(7/4)$.