Math 241, Spring 2007, Merit Worksheet 6

1. Find the curvature κ, the unit tangent vector $\mathbf{T}$, the unit normal vector $\mathbf{N}$, the tangential and normal components of acceleration for the space curve with position vector $\gamma(t) =< t, t^2, t^3 >$ at the point $(1, 1, 1)$.

2. Describe and sketch the graph of the equation $4x^2 - y^2 + 9z^2 = 1$. What are the traces of this surface in planes parallel to the coordinate axes? What name is given to a surface of this shape?

3. **Polar Coordinates:** Which of the following regions resembles a quarter of a doughnut?

   (a) $0 \leq r \leq 5$, $0 \leq \theta \leq \pi/2$

   (b) $3 \leq r \leq 5$, $0 \leq \theta \leq 2\pi$

   (c) $3 \leq r \leq 5$, $\pi \leq \theta \leq 2\pi$

   (d) $3 \leq r \leq 5$, $\pi \leq \theta \leq 3\pi/2$

4. **Cylindrical Coordinates:**

   (a) Graph $r = 5$.

   (b) Graph $\theta = 3\pi/4$.

   (c) Graph $z = 7\pi/4$.

   (d) Mark the point $(5, 3\pi/4, 7\pi/4)$. What are its cartesian coordinates?

5. Which of the following regions represents the portion of a cylinder of height 4 and radius 3 above the $3rd$ quadrant of the xy plane?

   (a) $1 \leq r \leq 3$, $0 \leq z \leq 4$, $0 \leq \theta \leq \pi/2$

   (b) $0 \leq r \leq 4$, $0 \leq z \leq 4$, $\pi \leq \theta \leq 3\pi/2$

   (c) $0 \leq r \leq 4$, $0 \leq z \leq 3$, $\pi \leq \theta \leq 3\pi/2$

   (d) $0 \leq r \leq 3$, $0 \leq z \leq 4$, $0 \leq \theta \leq \pi/2$

6. **Spherical Coordinates:**

   (a) Graph $\rho = 5$.

   (b) Graph $\phi = 3\pi/4$.

(c) Graph $\theta = 7\pi/4$.

(d) Mark the point $(5, 3\pi/4, 7\pi/4)$. What are its cartesian coordinates?

(e) Graph $\phi = \pi/2$.

7. Convert the equation into both cylindrical and spherical coordinates:

   (a) $x^2 + y^2 = 2x$

   (b) $z = x^2 - y^2$

8. Describe the graph of the equation:

   (a) $\rho = 4\cos\phi$.

   (b) $\rho^3 - 4\rho = 0$.

9. Write an equation for the surface generated by revolving this curve around the indicated axis. Then sketch the surface:

   (a) The line $z = 2x$; the x-axis.

   (b) The line $z = 3x$; the z-axis.

   (c) $x = 2y^2$; the y-axis.

10. Find the domains of the following functions (on $\mathbb{R}^3$):

    (a) $f(x, y, z) = \sqrt{x - y}$

    (b) $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$

    (c) $f(x, y, z) = \frac{\log xyz}{xy^2 - xy}$

    (d) $f(x, y, z) = 4x^2y^4z^8 + z^2 + \sqrt{1 + x^2}$

    Find where the function in (a) has value 4. Find where the function in (b) has value 0. Find where the function in (d) has value -1.

Warm-Up Problems for Next Time

1. Find the largest possible domain of definition for the function

$$f(x, y) = \frac{xy}{x^2 - y^2}$$

2. Find $\lim_{(x,y)\to(0,0)} \frac{\cos(x^2+y^2)}{1-x^2-y^2}$.