

Math 241, Spring 2007, Merit Worksheet 7

1. Convert the equation into both cylindrical and spherical coordinates:

(a) $x^2 + y^2 = 2x$

(b) $z = x^2 - y^2$

2. Describe the graph of the equation:

(a) $\rho = 4 \cos \phi$.

(b) $\rho^3 - 4\rho = 0$.

3. Write an equation for the surface generated by revolving this curve around the indicated axis. Then sketch the surface:

(a) The line $z = 3x$; the z -axis.

(b) $x = 2y^2$; the x -axis.

4. Find the domains of the following functions (on \mathbb{R}^3):

(a) $f(x, y, z) = \sqrt{x - y}$

(b) $f(x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$

(c) $f(x, y, z) = \frac{\log xyz}{xy^2 - xy}$

(d) $f(x, y, z) = 4x^2y^4z^8 + z^2 + \sqrt{1 + x^2}$

Find where the function in (a) has value 4. Find where the function in (b) has value 0. Find where the function in (d) has value -1 .

5. The diagram below shows the level curves of a function.

What path will result in the greatest change in altitude? Which path is the steepest?

(a) A to B

(b) A to C

(c) A to D

(d) All the same.

6. Can you think of two or more surfaces which have the following as their level curves?
7. Sketch some typical level curves of the function $f(x, y) = y - x^2$.
8. What are the cylindrical coordinates of a sphere centred at $(0, 0, 2)$ of radius 3?
9. The angle between the vectors $-x\vec{i} - \vec{j} + \vec{k}$ and $x\vec{i} + 2\vec{j} - 3\vec{k}$:
 - (a) is between 0 and 45 degrees
 - (b) is between 45 and 90 degrees
 - (c) is greater than 90 degrees
 - (d) can be any of the above depending on the value of x .
10. Two vectors have a dot product of 14. To guarantee the dot product is equal to 28, you could:
 - (a) double the angle between the vectors
 - (b) double the length of both vectors
 - (c) double the length of one vector
 - (d) none of the above

Warm-Up Problems for Next Time

1. I'll hold a review session/practice exam on Saturday at 3pm. It will be in the Merit Room.
2. Take the exam Monday at 9. Best of luck!