Math 241, Spring 2007, Merit Worksheet 8

1. Determine whether the limits below exist:

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$$

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^2+z^2}$$

$$\lim_{(x,y,z,w)\to(0,0,0,0)} \frac{x^2+y^2+z^2-w^2}{x^2+y^2+z^2+w^2}$$

2. Discuss the continuity of the function

$$h(x,y,z) = \begin{cases} \frac{\sin(x^2-y^2)}{x^2-y^2} & \text{unless } x^2=y^2 \\ 1 & \text{if } x^2=y^2 \end{cases}$$

3. If $f(x,y)=x^y$, find $f_x(x,y)$ and $f_y(x,y)$.

4. Suppose $f_x(x,y)=y$ and $f_y(x,y)=x+y$. What is $f(x,y)$? What about $g_x(x,y)=x+4y$ and $g_y(x,y)=3x-y$?

5. Let $f(x,y,z,w)=w^2\sin(x^2+y^2+z^2)$. Find

$$f_{xyzw}(\sqrt{\pi/12},\sqrt{\pi/12},\sqrt{\pi/6},1) \qquad \text{and} \qquad \left.\frac{\partial^2 f}{\partial x\partial w}\right|_{0,0,\sqrt{\pi/4},2}.$$

6. Consider $f(x,y,z)=x^2\cos(y^3+z^2)$.

   (a) Why do we know that $f_{zyyxxx}=0$ without doing any computations?

   (b) Do we also know, without any computations, that $f_{xyzzz}=0$?

7. What vector is normal to the surface $z=f(x,y)=5x^2+7y^3+2x+3y+6$ at the point $(0,0,6)$?

8. Which of the following could be the equation of the tangent plane to the surface $z=x^2+y^2$ at a point (a,b) in the first quadrant?

   (a) $z=-3x+4y+7$

(b) $z = 2x - 4y + 5$

(c) $z = 6x + 6y - 18$

(d) $z = -4x - 4y + 24$

9. Find the one point at which the plane tangent to the surface

$$z = x^2 + 2xy + 2y^2 - 6x + 8y$$

is horizontal.

10. Let $f(x, y) = \frac{2x^2y}{x^4+y^2}$.

    (a) Show that $f(x, y) \to 0$ as $(x, y) \to (0, 0)$ along any and every straight line through the origin.

    (b) What happens as $(x, y) \to (0, 0)$ along the parabola $y = x^2$?

    (c) What is $\lim_{(x,y)\to(0,0)} f(x, y)$?

Warm-Up Problems for Next Time

1. Find every point on the surface $z = x^2 + 4x + y^3$ at which the tangent plane is horizontal.