Math 241, Spring 2007, Merit Worksheet 9

1. Consider $f(x, y, z) = x^2 \cos(y^3 + z^2)$.

   (a) Why do we know that $f_{zyyxxx} = 0$ without doing any computations?

   (b) Do we also know, without any computations, that $f_{xyzzz} = 0$?

2. What vector is normal to the surface $z = f(x, y) = 5x^2+7y^3+2x+3y+6$ at the point $(0, 0, 6)$?

3. Find the one point at which the plane tangent to the surface

   $$z = x^2 + 2xy + 2y^2 - 6x + 8y$$

   is horizontal.

4. Find the equation of the tangent plane to the surface $z = x^2 - 4y^2$ at the point $(5, 2, 9)$.

5. You are standing at the point where $x = y = 100$ ft on a hillside whose height (in feet above sea level) is given by

   $$z = 100 + \frac{1}{100}(x^2 - 3xy + 2y^2),$$

   with the positive x-axis to the east and the positive y-axis to the north.

   (a) If you head due east, will you initially be ascending or descending? At what angle in degrees from the horizontal?

   (b) If you head due north, will you initially be ascending or descending? At what angle in degrees from the horizontal?

6. Show that the function

   $$u = u(x, t) = e^{-n^2kt} \sin nx$$

   satisfies the one-dimensional heat equation

   $$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}.$$

   (k and n are constants.)

7. Find the differential $\mathrm{d}f$ of the function $f(x, y, z) = x^2y + xyz + z^2 + 4$.

8. Suppose $f_x(3, 4) = 5$, $f_y(3, 4) = -2$, and $f(3, 4) = 6$. Assuming the function is differentiable, what is the best estimate for $f(3.1, 3.9)$ using this information?

9. Use differentials to approximate $\sqrt{26}\sqrt[3]{28}\sqrt[4]{17}$.

10. Use differentials to approximate $e^{0.4} = e^{(1.1)^2-(0.9)^2}$.

    Now do this problem by finding the tangent plane to a surface.

11. The base radius r and the height h of a right circular cylinder are measured as 3 cm and 9 cm, respectively. There is a possible error of 1 mm in each measurement. Use differentials to estimate the maximum possible error in computing:

    (a) the volume of the cylinder;

    (b) the total surface area of the cylinder.

Warm-Up Problems for Next Time

1. Find $\frac{\mathrm{d}w}{\mathrm{d}t}$ both by using the chain rule and by expressing w as a function of t before differentiating, where

$$w = \frac{1}{u^2 + v^2}; \qquad u = \cos 2t; \qquad v = \sin 2t$$