

Math 225 Section P2: Introductory Matrix Theory
Final

Name:

- Please explain your work. Simply saying 'yes' or 'no' is not sufficient.
- When performing matrix operations, give some indication of how you get from one matrix to another: (multiply row 1 by 5, add row 2 to row 1, etc.)
- If you have extra time, please check your work.
- NO CALCULATORS ALLOWED.

Problem	Possible	Received
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
Total	200	

1. Find the general solution to the following system of linear equations and write the solution in parametric vector form.

$$x_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 2 \\ 5 \\ 8 \end{bmatrix} + x_4 \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \\ 12 \end{bmatrix}$$

2. (a) Is the following set of vectors linearly independent? $\left\{ \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} \right\}$

(b) Does this set of vectors span \mathbb{R}^3 ? $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$

(c) Are these vectors linearly independent? $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \right\}$

3. (a) Given the matrices, A , B and C , compute B^T , $A * B$, $C * B$ and AI_2 if they exist. If they do not exist, explain why.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -1 \\ 1 & 3 \\ 0 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

(b) Find the inverses of A and C .

4. Find the determinant of the following matrices.

$$A = \begin{bmatrix} 0 & 2 & 0 & 0 \\ 0 & 4 & -3 & 2 \\ 1 & 6 & 1 & -5 \\ 1 & -7 & 3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Do the columns of A form a basis of \mathbb{R}^4 ? How many solutions does the equation $B\mathbf{x} = \mathbf{0}$ have?

5. (a) Is \mathbb{R}^2 a subspace of \mathbb{R}^3 ? Why or why not?

(b) If W is the set of all polynomials of the form $a + bt + ct^2$ where a , b , and c are elements of \mathbb{R} , is W a subspace of \mathbb{P}_2 (the set of all polynomials of degree ≤ 2)?

(c) If $V = \left\{ \begin{bmatrix} 4s + 2t \\ 7s - t \\ -3s + t \\ 1 \end{bmatrix} : s, t \in \mathbb{R} \right\}$, is V a subspace of \mathbb{R}^4 ?

6. (a) Assume A is row equivalent to B.

$$A = \begin{bmatrix} 1 & 0 & 2 & 1 & 5 \\ 1 & 1 & 5 & 2 & 7 \\ 1 & 2 & 8 & 4 & 12 \\ 0 & 1 & 3 & 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 2 & 0 & 2 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Find a basis for Nul A, Col A and Row A. Determine Rank A and dim Nul A.

(b) If A is a 5×3 matrix, what is the largest possible rank of A ? How about if A is 3×5 ? Explain your answer.

7. (a) Is $\lambda = 3$ an eigenvalue of $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$? Is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ an eigenvector?

(b) If $A = \begin{bmatrix} -2 & 10 & 5 \\ 0 & 3 & 2 \\ 0 & 0 & -2 \end{bmatrix}$, find the characteristic equation, eigenvalues and bases for the eigenspaces of A.

(c) Find the eigenvalues of $A = \begin{bmatrix} 5 & -4 \\ 3 & -3 \end{bmatrix}$ and then diagonalize A (i.e. find P , D and P^{-1}).

8. Determine if each of the sets of vectors in problem 2 is a basis for \mathbb{R}^n for some n . Say why or why not!

9. (a) Let $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 5 \\ -4 \\ -2 \end{bmatrix}$. Find the length of \mathbf{u} and $dist(\mathbf{u}, \mathbf{v})$.
Are these two vectors orthogonal? Normalize \mathbf{u} .

(b) Let $S = \left\{ \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix} \right\}$. Is this an orthogonal set? Is it orthonormal? Explain your answer.

10. Let $\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Find the orthogonal projection of \mathbf{y} onto $\text{Span}\left\{ \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \right\}$.

Bonus