Theorem 1. *If $B = \{\vec{b_1}, \vec{v_2}, \ldots, \vec{v_n}\}$ is a basis for the vector space V, then any set in V containing more than n vectors must be linearly independent.*

Theorem 2. *If a vector space V has a basis of n vectors, then every basis of V has precisely n vectors.*

Definition 1. *The dimension of a vector space V is the number of vectors in any basis for V (may be infinite). Define the dimension of the trivial subspace (i.e,. $\{0\}$) to be zero.*

Theorem 3. *Let H be a subspace of a finite-dimensional vector space V. Any linearly independent set in H can be expanded, if necessary, to a basis for H. In such a case, H is finite-dimensional and*

$$\dim H \leq \dim V.$$

Theorem 4 ((**The basis Theorem**))**.** *Let V be an n-dimensional vector space with $n \geq 1$. Any linearly independent set of exactly n elements in V is a basis for V. Also, any set of exactly n elements that spans V is a basis for V.*

Definition 2. *Define the row space of A to be the column space of A^T. Equivalently, the row space is the set of all linear combinations of the rows of A. Denote the row space of A by Row A.*

Theorem 5. *If A is row-equivalent to B, then Row $A =$ Row B.*

Definition 3. *The rank of A is the dimension of the column space of A, denoted Rank A.*

Theorem 6. *If A is an $n \times m$ matrix then*

$$\dim(Row\ A) = \dim(Col\ A) = Rank\ A$$

and

$$Rank\ A + \dim(Nul\ A) = n.$$

Theorem 7 ((**The Invertible Matrix Theorem**) (Addendum))**.** *Let A be an $n \times n$ matrix. The following are equivalent to the statement "A is an invertible matrix":*

(1) *The columns of A form a basis for $\mathbb{R}^n$,*
(2) $Col\ A = \mathbb{R}^n$,
(3) $\dim\ (Col\ A) = n$,
(4) $Rank\ A = n$,
(5) $Nul\ A = \{\vec{0}\}$,
(6) $\dim(Nul\ A) = 0$