

Name: _____ key _____

Quiz 10

Justify all your work. Partial credit will be given if you show your reasoning.

- (1) Suppose A and B are $n \times n$ matrices. Use properties of determinants to answer the following (give reasons for your answers):

The key in answering each of these questions is that $\det(AB) = \det A \det B$, and a matrix is noninvertible if and only if its determinant is zero.

- (a) If A and B are singular, is AB singular or nonsingular?

In this case,

$$\det A = 0 \quad \text{and} \quad \det B = 0.$$

So

$$\det AB = \det A \cdot \det B = 0 \cdot 0 = 0.$$

Therefore AB is singular.

- (b) If A is a singular and B is a nonsingular, is AB singular or nonsingular?

In this case,

$$\det A = 0 \quad \text{and} \quad \det B \neq 0.$$

So

$$\det AB = \det A \cdot \det B = 0 \cdot \det B = 0.$$

Therefore AB is singular.

- (c) If A is nonsingular and B is nonsingular, is AB singular or nonsingular?

In this case,

$$\det A \neq 0 \quad \text{and} \quad \det B \neq 0.$$

So

$$\det AB = \det A \cdot \det B \neq 0.$$

Therefore AB is nonsingular.