MATH 225 March 26, 2002

Name: key

Quiz 11

Justify all your work. Partial credit will be given if you show your reasoning.

(1) Determine if $\vec{w} = \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix}$ is in $Nul\ A$, where $A = \begin{bmatrix} 5 & 21 & 19 \\ 13 & 23 & 2 \\ 8 & 14 & 1 \end{bmatrix}$.

Recall that $Nul\ A$ is the solution set to the homogeneous system $A\vec{x} = \vec{0}$. Thus, we only need to see whether $\vec{w}$ satisfies the homogeneous equation $A\vec{x} = 0$; i.e., is $A\vec{w} = \vec{0}$? We find

$$A\vec{w} = \begin{bmatrix} 5 & 21 & 19 \\ 13 & 23 & 2 \\ 8 & 14 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Thus, $\vec{w}$ is in $Nul\ A$.

(2) Show that the set $H = \left\{ \begin{bmatrix} a \\ 0 \end{bmatrix} : a \in \mathbb{R} \right\}$ is a subspace of $\mathbb{R}^2$. [Hint: you need to show three things].

(a) *Letting $a = 0$, we see that the zero vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ of $\mathbb{R}^2$ is in H.*

(b) *Let $\begin{bmatrix} b \\ 0 \end{bmatrix}$ and $\begin{bmatrix} c \\ 0 \end{bmatrix}$ be two vectors in H. Then*

$$\begin{bmatrix} b \\ 0 \end{bmatrix} + \begin{bmatrix} c \\ 0 \end{bmatrix} = \begin{bmatrix} b+c \\ 0 \end{bmatrix}$$

is in H since $b + c$ is a real number.

(c) *Let $\begin{bmatrix} b \\ 0 \end{bmatrix}$ be a matrix in H and r be a real number. Then*

$$r \begin{bmatrix} b \\ 0 \end{bmatrix} = \begin{bmatrix} rb \\ 0 \end{bmatrix}$$

is in H since rb is a real number.

Since the zero vector of $\mathbb{R}^2$ is in H and since we have closure under the operations of addition and scalar multiplication (the sum of two vectors in H, and a scalar multiple of a vector in H is in H), H is a subspace of $\mathbb{R}^2$.